Ant-Q

A Reinforcement Learning approach to the traveling salesman problem

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Background

The ideas behind the Ant-Q Algorithm

Ant System

- A colony of cooperating ants leaving pheromone trails on the paths to find food.
- Random exploration → following paths with familiar pheromones.
- Pheromones can evaporate as time passes.
- Paths that are shorter will have less pheromone evaporation.

Q-learning

- Reinforcement Learning with states, rewards, and actions.
    - Finite states
    - Finite actions
- Model-free environment - interacting directly with the environment to find optimal policy instead of creating a model.
- Trial-and-error - does many trials and updates its policy as it learns
- Q-values: Q(s, a) - evaluation of the quality of action a in state s.
    - Current estimate of sum of future rewards if we take action a.
- Q-table: gives Q-values for every action in every state.
    - Rows: states
    - Columns: actions
    - Use TDs to update previous Q-values after evaluating current state/actions.

Q-learning

Temporal Difference

The reward received for the action taken in the previous state

The largest Q-value available for any action in the current state (the largest predicted sum of future rewards)

$$TD(s_t, a_t) = r_t + \gamma \cdot \max_a Q(s_{t+1}, a) - Q(s_t, a_t)$$

Temporal difference for the action taken in the previous state

The discount factor (between 0 and 1)

The Q-value for the action taken in the previous state

Bellman Equation

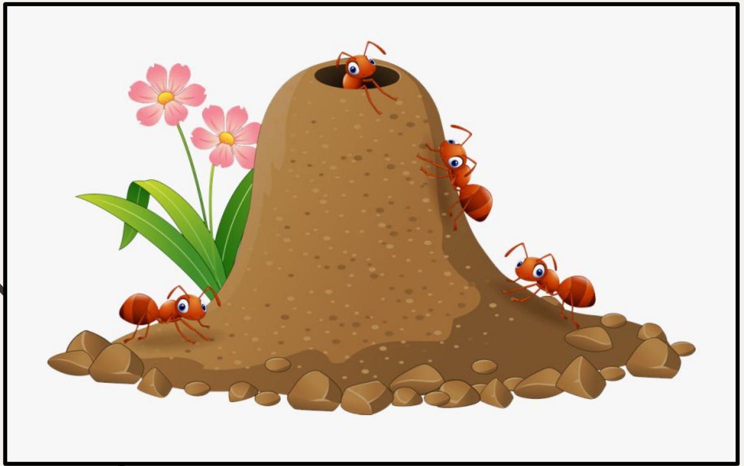
The old Q-value for the action taken in the previous state

The temporal difference for the action taken in the previous state

$$Q^{new}(s_t, a_t) = Q^{old}(s_t, a_t) + \alpha \cdot TD(s_t, a_t)$$

The new Q-value for the action taken in the previous state

The learning rate (between 0 and 1)



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Ant-Q

The Ant-Q algorithm

Ant-Q Algorithm

Travelling Salesman Problem

- Goal: find a minimal length closed tour that visits each city once
- *n* cities
- Each pair of cities has distance d_{rs}
- Connected graph with (N,E), N = set of n nodes, E = set of edges between cities
- HE(r,s) - heuristic evaluation of edge (r,s) - inverse of the distance
- AQ(r,s) - How useful it is to go to city s when at city r

Ant-Q Algorithm

Action Choice Rule

- Agent k - makes a tour
- Has a list $J_k(r)$ of cities that need to be visited. r = current city

$$s = \begin{cases} \arg\max_{u \in J_k(r)} \left\{ [AQ(r,u)]^{\delta} \cdot [HE(r,u)]^{\beta} \right\} & \text{if } q \le q_0 \\ S & \text{otherwise} \end{cases}$$

Ant-Q Algorithm

AQ-value Updates

- Similar to updating Q-values in Q-learning
- Includes delayed reinforcement value ΔAQ(r,s)

$$AQ(r,s) \leftarrow (1-\alpha)\cdot AQ(r,s) + \\ +\alpha \cdot \left(\Delta AQ(r,s) + \gamma \cdot \underset{z \in J_k(s)}{Max} AQ(s,z) \right)$$

Ant-Q Algorithm Steps

Step 1: Initialize

1. AQ-values
2. Multiple agents, each agent is placed on a city
3. $J_k(r_{k1})$ - set of cities that need to be visited

Step 2: Cycle

1. Each agent makes a move
2. AQ(r,s)'s are updated

Step 3: Delayed Reinforcement

1. Length L_k of each agent's tour is computed
2. Use lengths to compute delayed reinforcements
3. AQ(r,s)'s are updated with delayed reinforcements

Step 4: Termination Check

1. Check if termination condition is met
2. If not, return to step 2.

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Structural Parameters

The Action Choice Rule and Delayed Reinforcement

Action-Choice Rule

- Pseudo-random
- Pseudo-random-proportional
- Random-proportional

Delayed Reinforcement

- Global-best
- Iteration-best

The Action-Choice Rule

$$s = \begin{cases} \arg\max_{u \in J_k(r)} \left\{ [AQ(r,u)]^{\delta} \cdot [HE(r,u)]^{\beta} \right\} & \text{if } q \leq q_0 \\ S & \text{otherwise} \end{cases}$$

(1)

$$p_k(r,s) = \begin{cases} \dfrac{[AQ(r,s)]^{\delta} \cdot [HE(r,s)]^{\beta}}{\sum_{u \in J_k(r)} [AQ(r,u)]^{\delta} \cdot [HE(r,u)]^{\beta}} & \text{if } s \in J_k(r) \\ 0 & \text{otherwise} \end{cases}$$

(2)

All use (1) to determine which city to go next.

Pseudo-random Rule

- Uniform Distribution

Pseudo-random-proportional Rule

- The distribution in (2)

Random-proportional Rule

- Same as Pseudo-random-proportional, but with q_0 = 0. The choice of the next city is random, chosen with distribution in (2).

The Action-Choice Rule

	Pseudo-random				Pseudo-random-proportional				Random-proportional			
	γ	mean	std dev	best	γ	mean	std dev	best	γ	mean	std dev	best
City Set 1	0.5	6.18	0.06	6.03	0.3	5.87	0.05	5.84	0.9	7.85	0.25	7.40
City Set 2	0.5	6.26	0.04	6.20	0.3	6.06	0.05	5.99	0.9	7.77	0.30	7.43
City Set 3	0.5	5.69	0.07	5.61	0.3	5.57	0.00	5.57	0.9	7.89	0.17	7.75
City Set 4	0.5	5.92	0.05	5.84	0.3	5.76	0.03	5.70	0.9	7.95	0.10	7.85
City Set 5	0.5	6.30	0.04	6.22	0.3	6.18	0.01	6.17	0.9	8.48	0.21	8.10
Oliver30	0.5	425.02	1.22	424.69	0.3	424.44	0.46	423.74	0.9	515.19	10	493.20
ry48p	0.3	15602	440	14848	0.3	14690	175	14422	0.9	19495	797	17921

Delayed Reinforcement

Global-best

- Globally best tour from the beginning of the trial.
- Only the AQ-values for edges in the globally best tour will be reinforced.

$$\Delta AQ(r,s) = \begin{cases} \dfrac{W}{L_{k_{gb}}} & \text{if } (r,s) \in \text{tour done by agent } k_{gb} \\ 0 & \text{otherwise} \end{cases}$$

(1)

Iteration-best

- Best tour in the current iteration of the trial.
- Slightly faster with same quality.
- Less sensitive to changes of discount factor γ.

$$\Delta AQ(r,s) = \begin{cases} \dfrac{W}{L_{k_{ib}}} & \text{if } (r,s) \in \text{tour done by agent } k_{ib} \\ 0 & \text{otherwise} \end{cases}$$

(2)

The Action-Choice Rule

	Ant-Q Global-best			Ant-Q Iteration-best		
	mean	std. dev.	best	mean	std. dev.	best
City Set 1	5.90	0.08	5.84	5.87	0.05	5.84
City Set 2	6.05	0.04	5.99	6.06	0.05	5.99
City Set 3	5.58	0.01	5.57	5.57	0.00	5.57
City Set 4	5.76	0.03	5.70	5.76	0.03	5.70
City Set 5	6.20	0.03	6.17	6.18	0.01	6.17
Oliver 30	424.37	0.43	423.74	424.44	0.46	423.74
ry48p	14697	157	14442	14690	157	14422

Comparisons - Ant System

Delayed Reinforcement

$$\Delta AQ(r,s) = \sum_{k=1}^{m} \Delta AQ_k(r,s)$$

$$\Delta AQ_k(r,s) = \begin{cases} \frac{W}{L_k} & \text{if } (r,s) \in \text{tour done by agent k} \\ 0 & \text{otherwise} \end{cases}$$

AQ-value Updates

$AQ(r,s) \leftarrow (1 - \alpha) \times AQ(r,s) + \Delta AQ(r,s)$

- Applies to all edges
- Simulate pheromones and pheromone evaporation

	Ant-Q			Ant system		
	mean	std. dev.	best	mean	std. dev.	best
6x6 grid	360	0	360	360	0	360
Oliver 30	424.44	0.46	423.74	425.46	0.51	423.74
ry48p	14690	157	14422	14889	223	14803

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Results

Properties and Comparisons

Observations and Characteristics

Agents do not make the same tour.

- Agents do not converge to a common path.
- λ-branching factor - shows the dimension of the search space.
- Number of edges that have an AQ-value that is larger than $\lambda(AQ_{max}(r,s) - AQ_{min}(r,s)) + AQ_{min}(r,s)$.
- $0 \leq \lambda \leq 1$
- The search space is reduced, but agents continue to explore a subset of the search space.

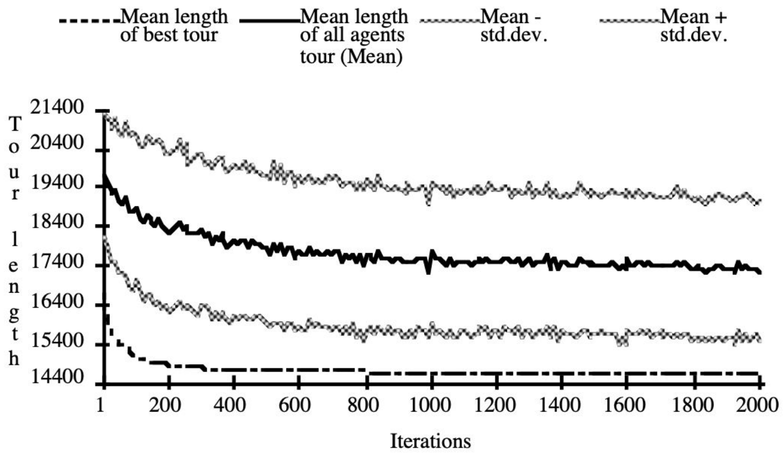

Figure 2: Mean length of best tour, mean length of all agents tour, and its std. dev. Problem: ry48p. Averaged over 15 trials.

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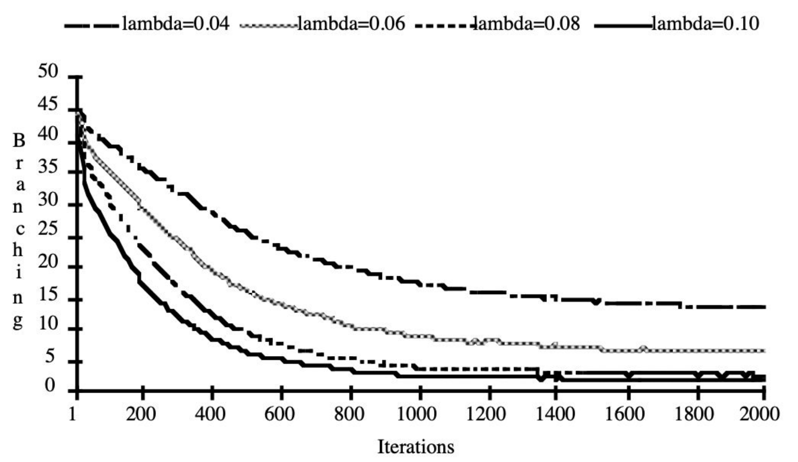

Figure 3: λ–branching factor. Problem: ry48p. Averaged over 15 trials.

Observations and Characteristics

AQ-values are exploited by agents to find short tours.

As iterations increase and good AQ-values are learned:

- AQ-values become more effective in finding good solutions.
- Heuristic values become less effective, even useless.

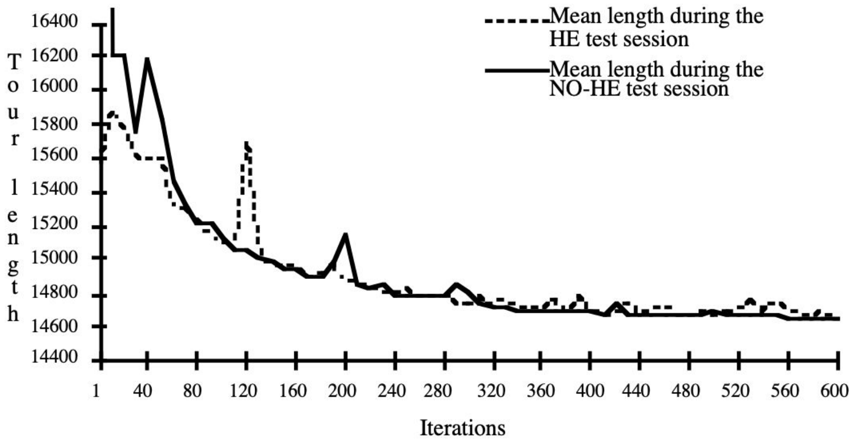

Figure 4: Best tour found during test session using only the AQ-values (NO-HE test session), and using both the AQ-values and the HE heuristic values (HE test session). The test session was run every ten learning iterations. Problem: ry48p. Averaged over 15 trials.

Comparison

Table 4: Comparisons on average result obtained on five 50-city problems. EN = elastic net, SA = simulated annealing, SOM = self organizing map, FI = farthest insertion, FI+2-opt = best solution found by FI and many distinct runs of 2-opt, FI+3-opt = best solution found by FI and many distinct runs of 3-opt. Results on EN, SA, and SOM are from Durbin and Willshaw (1989), and Potvin (1993). FI results are averaged over 15 trials starting from different initial cities. Ant-Q used pseudo-random-proportional action choice and iteration-best delayed reinforcement. It was run for 500 iterations and the results are averaged over 15 trials.

City set	EN	SA	SOM	FI	FI + 2-opt	FI + 3-opt	Ant-Q
1	5.98	5.88	6.06	6.03	5.99	5.90	**5.87**
2	6.03	**6.01**	6.25	6.28	6.20	6.07	6.06
3	5.70	5.65	5.83	5.85	5.80	5.63	**5.57**
4	5.86	5.81	5.87	5.96	5.96	5.81	**5.76**
5	6.49	6.33	6.70	6.71	6.61	6.48	**6.18**

Comparison

Table 5: Comparison between the best results obtained by SA+3-opt = best solution found by simulated annealing and many distinct runs of 3-opt, SOM+ = best solution found by SOM over 4,000 different runs (by processing the cities in various orders), FI and its locally optimized versions, and Ant-Q. The 2-opt and 3-opt heuristics used the result of FI as starting configuration for local optimization. Results on SA+3-opt and SOM+ are from Durbin and Willshaw (1989), and Potvin (1993). Ant-Q used pseudo-random-proportional action choice and iteration-best delayed reinforcement. It was run for 500 iterations, and the best result was obtained out of 15 trials.

City set	SA + 3-opt	SOM+	FI	FI + 2-opt	FI + 3-opt	Ant-Q
1	**5.84**	**5.84**	5.89	5.85	5.85	**5.84**
2	**5.99**	6.00	6.02	6.01	**5.99**	**5.99**
3	**5.57**	5.58	**5.57**	**5.57**	**5.57**	**5.57**
4	5.70	**5.60**	5.76	5.76	5.70	5.70
5	**6.17**	6.19	6.50	6.45	6.40	**6.17**

Comparison

Table 6: Comparison between exact methods and Ant-Q for difficult ATSP problems. Numbers in parenthesis are seconds. Type of delayed reinforcement: global-best. For the problem 43X2 we set γ=0.01. Ant-Q was run for 600 iterations, and results were obtained out of 15 trials.

Problem	*FT-92*	*FT-94*	*Ant-Q Mean*	*Ant-Q Best result*
ry48p	14422 (729.6)	14422 (52.8)	14690 (1590)	14422 (696)
43X2	N/A	5620 (492.2)	5625 (550)	5620 (238)

Conclusions

- Making Ant-Q more like Q-learning.
- Extend and apply Ant-Q to other combinatorial optimization problems.

N-Stroll

- Implement delayed reinforcement.
- Try using different action-choice rules.

Difficulties

- Implementing an ‘n’ requirement (could possibly use delayed reinforcement for that)
- Implementing different actions for different states (since each layer of nodes have different actions we can take).
- Whether or not we should implement multiple agents.

Works Cited

Dorigo M., V.Maniezzo and A.Colorni, 1996. The Ant System: Optimization by a colony of cooperating agents. *IEEE Transactions on Systems, Man, and Cybernetics*, 26, 2, in press.

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Thanks

Do you have any questions?