Cost of resizing

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Cost of additive upsizing – calculations

$$k = \left\lceil \frac{n - 10}{5} \right\rceil;$$

$$\sum_{i=0}^{k-1} (5 \times i + 10)$$

$$\frac{5}{2} \text{Ceiling}\left[\frac{1}{5} (-10 + n)\right] \left(3 + \text{Ceiling}\left[\frac{1}{5} (-10 + n)\right]\right)$$

Simplify[%]

$$\frac{5}{2} \left(-2 + \text{Ceiling}\left[\frac{n}{5}\right]\right) \left(1 + \text{Ceiling}\left[\frac{n}{5}\right]\right)$$

$$R^u (n) = \frac{5}{2} \left(-2 + \text{Ceiling}\left[\frac{n}{5}\right]\right) \left(1 + \text{Ceiling}\left[\frac{n}{5}\right]\right) \leq \frac{5}{2} \left(-2 + \frac{n+4}{5}\right) \left(1 + \frac{n+4}{5}\right)$$

$$\text{Simplify}\left[\frac{5}{2} \left(-2 + \frac{n+4}{5}\right) \left(1 + \frac{n+4}{5}\right)\right]$$

$$\frac{1}{10} (-6 + n) (9 + n)$$

$$\text{Expand}\left[\frac{1}{10} (-6 + n) (9 + n)\right]$$

$$-\frac{27}{5} + \frac{3 n}{10} + \frac{n^2}{10}$$

$$\text{Limit}\left[\frac{-\frac{27}{5} + \frac{3 n}{10} + \frac{n^2}{10}}{n^2}, n \to \infty\right]$$

$$\frac{1}{10}$$

$$R^u (n) = \frac{5}{2} \left(-2 + \text{Ceiling}\left[\frac{n}{5}\right]\right) \left(1 + \text{Ceiling}\left[\frac{n}{5}\right]\right) \geq \frac{5}{2} \left(-2 + \frac{n}{5}\right) \left(1 + \frac{n}{5}\right)$$

$$N\left[\frac{5}{2} \left(-2 + \frac{n}{5}\right) \left(1 + \frac{n}{5}\right)\right]$$

$$2.5 (-2. + 0.2 n) (1. + 0.2 n)$$

$$\text{Simplify}\left[\frac{5}{2} \left(-2 + \frac{n}{5}\right) \left(1 + \frac{n}{5}\right)\right]$$

$$\frac{1}{10} (-10 + n) (5 + n)$$

Expand$\left[\frac{1}{10}(-10+n)(5+n)\right]$

$$-5-\frac{n}{2}+\frac{n^2}{10}$$

Limit$\left[\frac{-5-\frac{n}{2}+\frac{n^2}{10}}{n^2},\ n\to\infty\right]$

$$\frac{1}{10}$$

Limit$\left[\frac{1}{n^2}\frac{5}{2}\left(-2+\text{Ceiling}\left[\frac{n}{5}\right]\right)\left(1+\text{Ceiling}\left[\frac{n}{5}\right]\right),\ n\to\infty\right]$

$$\frac{1}{10}$$

Average cost of upsizing per inserted element under assumption that no downsizing is ever made

$$\frac{R^u(n)}{n}\le\frac{\frac{1}{10}(-6+n)(9+n)}{n}$$

$$\frac{\frac{1}{10}(-6+n)(9+n)}{n}$$

Expand$\left[\frac{(-6+n)(9+n)}{10\ n}\right]$

$$\frac{3}{10}-\frac{27}{5\ n}+\frac{n}{10}$$

Limit$\left[\frac{\frac{3}{10}-\frac{27}{5\ n}+\frac{n}{10}}{n},\ n\to\infty\right]$

$$\frac{1}{10}$$

Limit$\left[\frac{1}{n}\frac{5}{2}\left(-2+\text{Ceiling}\left[\frac{n}{5}\right]\right)\left(1+\text{Ceiling}\left[\frac{n}{5}\right]\right)\Big/ n,\ n\to\infty\right]$

$$\frac{1}{10}$$

Cost of multiplicative upsizing – calculations

$$R^u(n)=\sum_{i=0}^{k-1}2^i\times 10$$

$$k = \left\lceil \text{Log2}\left[\frac{n}{10}\right] \right\rceil;$$

$$\sum_{i=0}^{k-1} 2^i \times 10$$

$$10\left(-1 + 2^{\text{Ceiling}\left[\frac{\text{Log}\left[\frac{n}{10}\right]}{\text{Log}[2]}\right]}\right)$$

$$R^u(n) = 10\left(-1 + 2^{\left\lceil \log_2\left[\frac{n}{10}\right] \right\rceil}\right)$$

$$\text{Since } \frac{n}{10} \le 2^{\left\lceil \log_2\left[\frac{n}{10}\right] \right\rceil} < 2\,\frac{n}{10}$$

$$10\left(-1 + \frac{n}{10}\right) \le R^u(n) < 10\left(-1 + 2\,\frac{n}{10}\right)$$

$$n - 10 \le R^u(n) < 2n - 10$$

Average cost of upsizing per inserted element under assumption that no downsizing is eved made

$$1 - \frac{10}{n} \le \frac{R^u(n)}{n} < 2 - \frac{10}{n}$$

$$\text{Limit}\left[1 - \frac{10}{n},\ n \to \infty\right]$$

1

$$\text{Limit}\left[2 - \frac{10}{n},\ n \to \infty\right]$$

2

$$\text{Limit}\left[\frac{10\left(-1 + 2^{\left\lceil \log_2\left[\frac{n}{10}\right] \right\rceil}\right)}{n},\ n \to \infty\right]$$

$$\text{Limit}\left[\frac{1}{n}10\left(-1 + 2^{\text{Ceiling}\left[\log_2\left[\frac{n}{10}\right]\right]}\right),\ n \to \infty\right]$$