Hashing

λ = the factor of table full

$f_u(\lambda)$ = expected number of probes for unsuccessful search

$f_s(\lambda)$ = expected number of probes for successful search

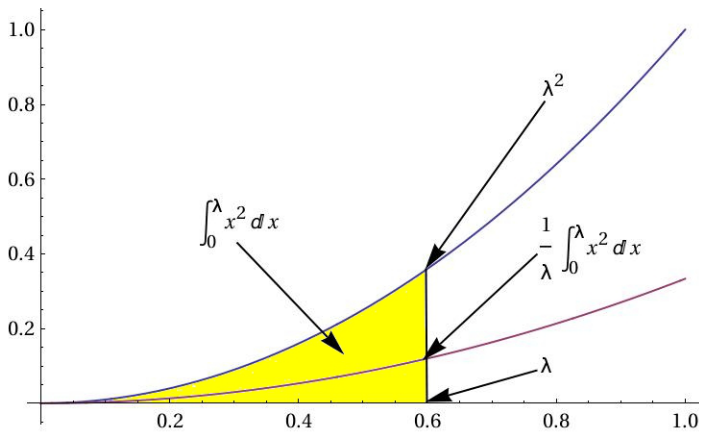
$$f_s(\lambda) = \frac{1}{\lambda}\int_0^{\lambda} f_u(x)\, dx$$

or (for separate chaining) $= \frac{1}{\lambda}\int_0^{\lambda} (1 + f_u(x))\, dx$

Comment : 1 in the above formula is the cost of retireving the list from the hash table.

Example $f_s(\lambda) = \lambda^2$

In[3]:= Plot$\left[\left\{\lambda^2, \frac{1}{\lambda}\int_0^{\lambda} x^2\, dx\right\}, \{\lambda, 0.0001, 1\}, \text{PlotTheme} \to \text{"Classic"}\right]$



Linear probing

Expected number of probes for unsuccessful search :

Ending up in an empty cluster :

1

Ending up in a non - empty cluster :

$$\sum_{i=1}^{\infty} i \times \lambda^{i-1}$$

$$\frac{1}{(-1+\lambda)^2}$$

Together, on average :

$$\frac{1}{2}\left(1 + \sum_{i=1}^{\infty} i \times \lambda^{i-1}\right)$$

$$\frac{1}{2}\left(1 + \frac{1}{(-1+\lambda)^2}\right)$$

Expected number of probes for successful search :

$$\frac{1}{\lambda}\int_0^{\lambda}\frac{1}{2}\left(1+\left(\frac{1}{1-x}\right)^2\right)dx$$

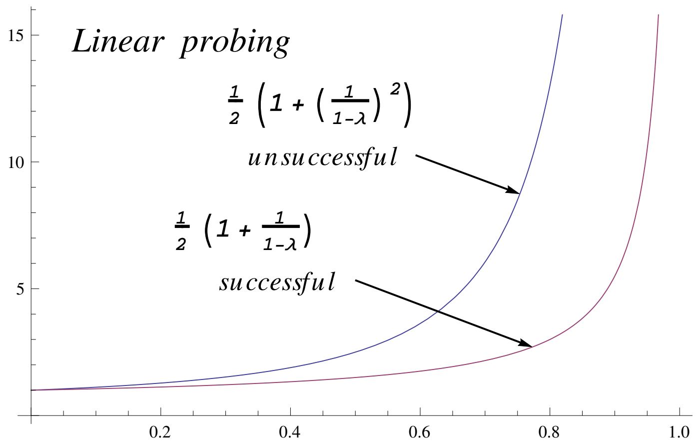
ConditionalExpression$\left[\frac{-2+\lambda}{2\,(-1+\lambda)}, \text{Re}[\lambda] \leq 1 \,||\, \lambda \notin \text{Reals}\right]$

$$\frac{-2+\lambda}{2\,(-1+\lambda)} == \frac{1}{2}\left(1+\left(\frac{1}{1-\lambda}\right)\right)$$

Simplify$\left[\frac{1}{2}\left(1+\left(\frac{1}{1-\lambda}\right)\right)\right]$

$$\frac{-2+\lambda}{2\,(-1+\lambda)}$$

Plot$\left[\left\{\frac{1}{2}\left(1+\left(\frac{1}{1-\lambda}\right)^2\right), \frac{1}{2}\left(1+\left(\frac{1}{1-\lambda}\right)\right)\right\}, \{\lambda, 0, .999\}\right]$



<u>Double hashing</u>

These derivations are based under (somewhat unrealistic) assumption that all consecutive probes are statistically independent.

Expected number of probes for unsuccessful search :

$$\frac{1}{1-\lambda}$$

Same as :

$$\sum_{i=1}^{\infty} i \times \left(\lambda^{i-1} - \lambda^i\right)$$

$$-\frac{1}{-1+\lambda}$$

Explanation :
If λ^n is the probability of making alt least n + 1 trials then $\lambda^{i-1} - \lambda^i$ is the probability of making exactly i trials.

Same as :

$\sum_{i=0}^{\infty} \lambda^i$

$\frac{1}{1-\lambda}$

Same as :

$\sum_{i=1}^{\infty} i \times \lambda^{i-1} \times (1-\lambda)$

$-\frac{1}{-1+\lambda}$

Explanation :
1 probe with probability of 1 - λ (hit an empy slot at the first trial)
2 probes wih probability λ × (1 - λ)
(hit an occupied slot at the first trial and an empty slot at the second trial)
3 probes wih probability λ^2 × (1 - λ)
(hit an occupied slot at the first 2 trials and an empty slot at the third trial)
...
i probes wih probability λ^{i-1} × (1 - λ)
(hit an occupied slot at the first i - 1 trials and an empty slot at the i - th trial)
Hence the term i × λ^{i-1} × (1 - λ) in the above summation.

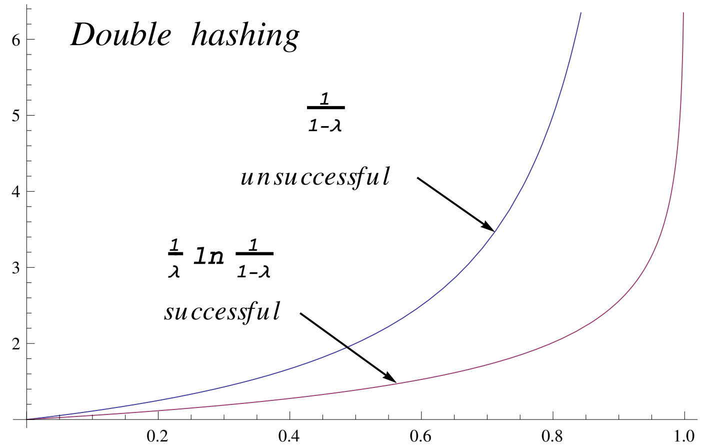
Expected number of probes for successful search :

$\frac{1}{\lambda}\int_0^{\lambda}\frac{1}{1-x}\,dx$

ConditionalExpression$\left[-\frac{\text{Log}[1-\lambda]}{\lambda}, \text{Re}[\lambda] \le 1 \,||\, \lambda \notin \text{Reals}\right]$

Expand$\left[-\frac{\text{Log}[1-\lambda]}{\lambda}\right]$

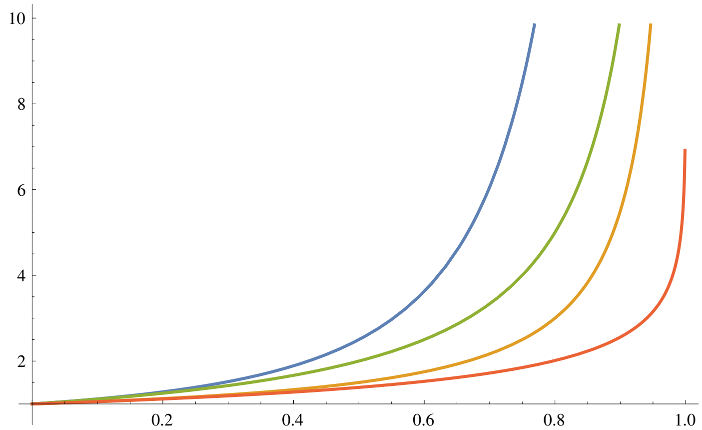
Plot$\left[\left\{\frac{1}{1-\lambda}, \frac{1}{\lambda}\text{Log}\left[\frac{1}{1-\lambda}\right]\right\}, \{\lambda, 0, .999\}\right]$



All four cases (linear probing and double hashing) :

In[27]:= Plot$\left[\text{Tooltip}\left[\left\{\frac{1}{2}\left(1+\left(\frac{1}{1-\lambda}\right)^2\right), \frac{1}{2}\left(1+\left(\frac{1}{1-\lambda}\right)\right), \frac{1}{1-\lambda}, \frac{1}{\lambda}\text{Log}\left[\frac{1}{1-\lambda}\right]\right\}\right], \{\lambda, 0, .999\}\right]$

Out[27]=



Separate chaining

Expected number of probes for unsuccessful search :

λ

Explanation :
λ is the average length of a list since there are B lists with a total of λB elements in them.

Expected number of probes for successful search :

$$\frac{1}{\lambda}\int_0^\lambda (1+x)\,dx$$

$$\frac{\lambda+\frac{\lambda^2}{2}}{\lambda}$$

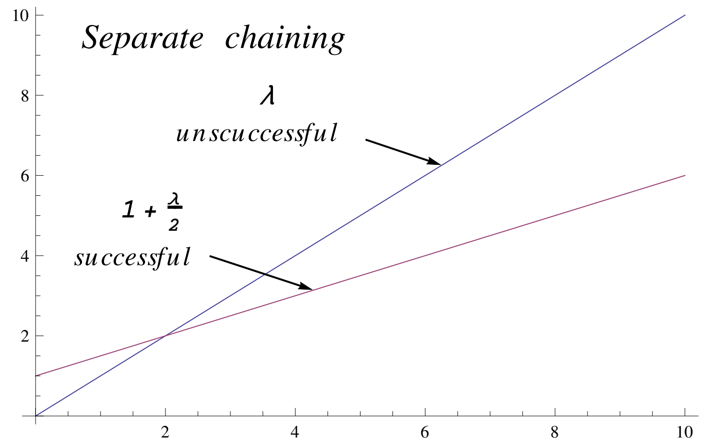
Simplify[%]

$$\frac{2+\lambda}{2}$$

Expand[%]

$$1+\frac{\lambda}{2}$$

Plot$\left[\left\{\lambda, 1+\frac{\lambda}{2}\right\}, \{\lambda, 0, 10\}\right]$


Separate chaining
λ
unscuccessful
1 + λ/2
successful
10
8
6
4
2
2
4
6
8
10