Maximum $ipl(n)$ and $epl(n)$ in any BS tree on n nodes

A binary tree T on n nodes has the maximum
internal path length among all binary trees on n nodes,

that is,

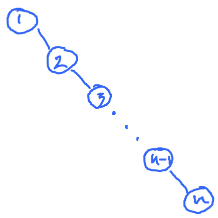
$ipl(T) = ipl_{max}(n)$

iff

T is empty (so, $n = 0$)

or

all levels of T have the minimum number of nodes (1 node each, that is).



Hence, the minimum internal path length
$ipl_{max}(n)$ is equal the sum of all consecutive integers from 0 to $n-1$.

Thus,

$$ipl_{max}(n) = \sum_{i=1}^{n} (i-1) = \frac{n(n-1)}{2}.$$

So,

$$ipl_{max}(n) = \frac{n(n-1)}{2}.$$

Note.
$\frac{n(n-1)}{2}$ is the number of comparisons of keys that Quicksort will perform
in the worst case while sorting an n – element array with no duplicate keys.

Therefore,

$$epl_{max}(n) = ipl_{max}(n) + 2n = \frac{n(n-1)}{2} + 2n = \frac{1}{2}(n(n-1) + 4n)$$
$$= \frac{n(n+3)}{2}.$$

or

$$epl_{min}(n) = \frac{n(n+3)}{2}$$

Average number of comparisonsfor successfulsearch in an average BS tree :

$$c_n = \frac{ipl_{avg}[n]}{n} + 1 \approx \frac{1}{n}\,\frac{1}{2}\,n\,(n-1) + 1 =$$

$$\frac{n-1}{2} + 1 = \frac{n+1}{2}.$$

$$c_n \approx \frac{n+1}{2}$$

Average number of comparisonsfor unsuccessfulsearch in an average BS tree :

$$c'_n = \frac{epl_{avg}[n]}{n+1} \approx \frac{n(n+3)}{2} \Big/ (n+1) =$$

$$\frac{n(n+3)}{2(n+1)} \approx \frac{n+3}{2}$$

$$c'_n \approx \frac{n+3}{2}.$$

NB*