The size of an integer

For in-class use only in CSC 311 course

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1 Binary representations of positive natural numbers

How many bits are needed to represent a number $M > 0$ in binary?

Say, it's n.

The greatest binary number that may be represented on n bits is n 1s

$$\underbrace{11\ldots1}_{n}.$$

The smallest binary number that needs n bits is one 1 followed by $n-1$ 0s

$$1\underbrace{00\ldots0}_{n-1} = \underbrace{100\ldots0}_{n}.$$

So, we have:

$$2^{n-1} = 1\underbrace{00\ldots0}_{n-1} = \underbrace{100\ldots0}_{n} \leq M \leq \underbrace{11\ldots1}_{n}.$$

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$$\underbrace{11\ldots1}_{n} + 1 = 1\,\underbrace{00\ldots0}_{n} = 2^n.$$

$$2^{n-1} \leq M \leq \underbrace{11\ldots1}_{n}.$$

$$\underbrace{11\ldots1}_{n}+1 = 1\underbrace{00\ldots0}_{n} = 2^n.$$

$$\underbrace{11\ldots1}_{n} = 2^n - 1.$$

$$2^{n-1} \leq M \leq \underbrace{11\ldots1}_{n}.$$

$$\underbrace{11\ldots1}_{n} = 2^n - 1.$$

$$2^{n-1} \leq M \leq 2^n - 1$$

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$$2^{n-1} \leq M < 2^n$$

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$$\log_2 2^{n-1} \leq \log_2 M < \log_2 2^n$$

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$$n - 1 \leq \log_2 M < n.$$

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$$n - 1 = \lfloor \log_2 M \rfloor$$

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$$n = \lfloor \log_2 M \rfloor + 1.$$

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So, $\lfloor \log_2 M \rfloor + 1$ bits are needed to represent number $M > 0$ in binary.

Here is an important equality that I recommend you try to memorize:

$$\lfloor \log_2 M \rfloor + 1 = \lceil \log_2(M + 1) \rceil,$$

for any integer $M \geq 1$.