1

$$E(f) = \sum_{e \in U} f(e) \times p(e)$$

$$E(f|s) = \sum_{e \in U} f(e) \times p(e|s) =$$

$$= \sum_{e \in U} f(e) \frac{p(\{e\} \cap S)}{p(S)} =$$

$$= \frac{1}{p(S)} \times \sum_{e \in U} f(e) \times p(\{e\} \cap S) =$$

$$= \frac{1}{p(S)} \times \sum_{e \in S} f(e) \times p(e)$$

<u>Thm.</u> Let V be a partition of U

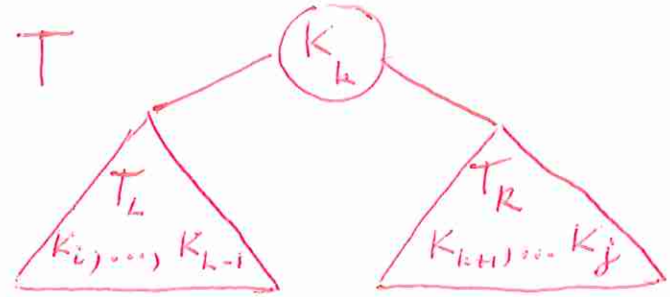
$$E(f) = \sum_{s \in V} p(s) \times E(f|s)$$

<u>Proof.</u> $\sum_{s \in V} p(s) \times E(f|s) =$

$$= \sum_{s \in V} p(s) \times \frac{1}{p(s)} \times \sum_{e \in s} f(e) \times p(e) =$$

$$= \sum_{s \in V} \sum_{e \in s} f(e) \times p(e) =$$

$$= \sum_{s \in V} f(e) \times p(e) = E(f) \quad \square$$



(2)

$$E(T) = p(T_L \mid T) \times (E(T_L) + 1) +$$

$$+ p(k \mid T) \times E(k) +$$

$$+ p(T_R \mid T) \times (E(T_R) + 1) =$$

$$= \frac{P(T_L)}{P(T)} \times (E(T_L) + 1) +$$

$$+ \frac{P_k}{P(T)} \times 1 +$$

$$+ \frac{P(T_R)}{P(T)} \times (E(T_R) + 1) =$$

$$= \frac{1}{P(T)} \Big(p(T_L) \times E(T_L) + P(T_L) +$$

$$+ P_k +$$

$$+ p(T_R) \times E(T_R) + P(T_R) \Big) =$$

$$= \frac{1}{p(i,j)} \Big(p(i,k-1) \times E(T_L) + p(i,k-1) + P_k +$$

$$+ p(k+1,j) \times E(T_R) + p(k+1,j) \Big) =$$

$$p(i,j) = \sum_{m=i}^{j} P_m = P(T); \quad p(i,k-1) = \sum_{m=i}^{k-1} P_m = P(T_L)$$

$$p(k+1,j) = \sum_{m=k+1}^{j} P_m = P(T_R)$$

(3)

$$= \frac{1}{p(i,j)} \times \big(p(i,k-1) \times E(T_L) +$$

$$+ p(k+1,j) \times E(T_R) + p(i,j)\big)$$

$\therefore\ p(i,j) \times E(T) = p(i,k-1) \times E(T_L) + p(k+1,j) \times E(T_R) + p(i,j).$

Putting:

$$A[i][j] = p(i,j) \times E(T)$$

$$A[i][k-1] = p(i,k-1) \times E(T_L)$$

$$A[k+1][j] = p(k+1,j) \times E(T_R)$$

we conclude:

$$A[i][j] = p(i,j) \times E(T) =$$

$$= p(i,k-1) \times E(T_L) + p(k+1,j) \times E(T_R) +$$
$$+ p(i,j) =$$

$$= A[i][k-1] + A[k+1][j] + p(i,j)$$

$\therefore\ A[i][j] = A[i][k-1] + A[k+1][j] + p(i,j)$

$P(i,k-1) + P_k + P(k+1,j) = \sum_{m=i}^{k-1} P_m + P_k + \sum_{m=k+1}^{j} P_m = \sum_{m=i}^{j} P_m = P(i,j)$

Because the choice of k was optimal, we get

$$A[i][j] = \min_{i \le k \le j} \left(A[i][k-1] + A[k+1][j] + p(i,j) \right) =$$

$$= p(i,j) + \min_{i \le k \le j} \left(A[i][k-1] + A[k+1][j] \right)$$

The above formula for $A[i][j]$ allows for computation of it using only indices p q such that $q - p < j - i$.