CSC 501/401 Analysis of Algorithms Spring '14

Practice Exam

DISCLAIMER

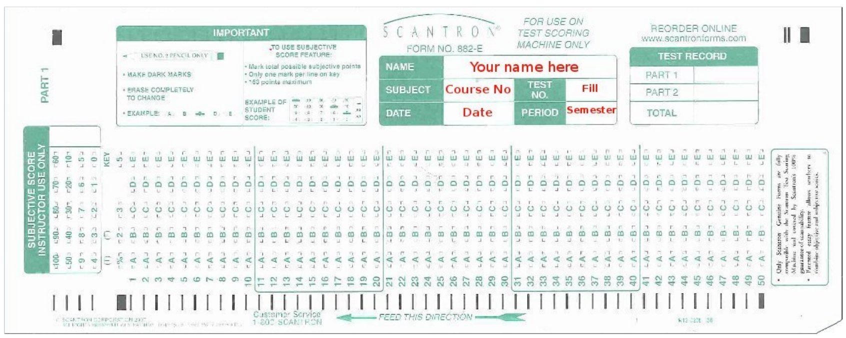
This practice exam does <u>*not*</u> *provide representative samples of actual questions on tests and final in this course. The number of questions on tests and final will be substantially greater than the number of questions on this practice exam. The students should not assume any similarity in scope, level of difficulty, and specificity between the questions on this practice exam and actual questions on tests and final.*

This is a closed textbook, no-notes test. You may only use this lab's workstation and the account provided to you for this test. The only software that you are allowed to use on this account during this test is Mathematica and Calculator. No access is allowed to your individual account, removable memory, the Internet, any other medium (including laptops, iPods, cellphones, etc.) or to any information pertaining to this course.

INSTRUCTIONS - READ CAREFULLY

There are 26 questions on this exam. Questions 1 through 22 are multiple-choice questions. Questions 23 through 26 are essay-type questions.

For all multiple-choice questions, use scantron form 882-E like this:



For all essay-type questions, use scratch paper provided by the professor. Do not use your own scratch paper for anything at any time.

Each multiple-choice question is worth $\frac{1}{2}$ point of credit for graduate students and 1 point of credit for undergraduate students. Each essay-type question is worth $2\frac{1}{2}$ points of credit for graduate students and 5 points of credit for undergraduate students. However, the maximum undergraduate credit for this exam is 30 points.

Pick one answer for each multiple-choice question and mark it clearly on your scantron. If none of the choices provided for any question seems correct, or more than one choice provided seems correct, chose the answer that, in your opinion, is the closest to the correct one or is the best one.

Multiple-choice questions are repeated as "Same as above" questions in order to allow partial credit for some partially correct answers. Although the correct answer is always the same for both, you may chose different answers for each of the two in the case you are not sure which one is correct. However, by doing so you will get less than the maximum score.

Write complete answers to essay-type questions on scratch paper provided by the professor. For full credit, show all your work.

THE QUESTIONS BEGIN HERE

PART I: MULTIPLE-CHOICE QUESTIONS

1. Give a closed-form formula for the following long summation:

$$\sum_{i=1}^{n} i \times 2^i.$$

   (A) $n \times 2^{n+1} - 1$
   (B) $(n-1)2^{n+1} + 2$
   (C) $(n+1)2^{\lfloor \lg n \rfloor + 1} + 2$
   (D) $\lceil \lg(n+1) \rceil$

2. Same as above.

3. What is the least power of 2 greater than n?

   (A) $\lceil x \rceil$, where $n \leq x < n+1$
   (B) $2^{\lceil x \rceil}$, where $n \leq x < n+1$
   (C) $2^{\lfloor \log_2 n \rfloor + 1}$
   (D) $2^{\lceil \log_2 n \rceil - 1}$

4. Same as above.

5. Which expression defines the *conditional expected value* (a.k.a. *conditional expectation*) $E(f|S)$ of random variable f under condition S?

   (A) $E(f) \times Pr(S)$
   (B) $\sum_{e \in S} f(e) \times Pr(e)$
   (C) $\sum_{e \in S} f(e) \times Pr(e|S)$
   (D) $\sum_{e \in U} f(e) \times Pr(e)$

   where U is the set of all elementary events and $Pr(x)$ is probability of x.

6. Same as above.
7. Which of the following inequalities is true for every non-increasing integrable function f, assuming $a+1<b$?

   (A) $\int_{a-1}^{b} f(x)dx \geq \sum_{n=a}^{b} f(n) \geq \int_{a}^{b+1} f(x)dx$
   (B) $\int_{a}^{b-1} f(x)dx \geq \sum_{n=a}^{b} f(n) \geq \int_{a+1}^{b} f(x)dx$
   (C) $\int_{a}^{b} f(x)dx \geq \sum_{n=a}^{b} f(n) \geq \int_{a+1}^{b+1} f(x)dx$
   (D) $\int_{a-1}^{b-1} f(x)dx \geq \sum_{n=a}^{b} f(n) \geq \int_{a}^{b} f(x)dx$

8. Same as above.
9. Let $f(n) = (n\log_2 n)^{1.5}$ and $g(n) = n^{1.56} + 0.5\sqrt{\log_2 n}$. Which of the following is true?

   (A) $g \in O(f)$
   (B) $f \in \Theta(g)$
   (C) $g \in o(f)$
   (D) $f \in o(g)$

10. Same as above.
11. What is the worst-case number of comparisons of keys performed by unsuccessful sequential search on an n-element ordered array?

    (A) n
    (B) $\frac{n+1}{2}$
    (C) $\lfloor \frac{n+1}{2} \rfloor$
    (D) $\frac{n}{n+1} + \frac{n}{2}$

12. Same as above.
13. What is the average-case number of comparisons of keys performed by successful binary search on an n-element ordered array?

    (A) $\lfloor \frac{n}{2} \rfloor$
    (B) $\lfloor \log_2 n \rfloor + 1$

(C) $\log_2(n+1) - 1 + o(1)$
(D) $n - 1$

14. Same as above.

15. What is the average number of comparisons of keys that `InsertionSort` performs while sorting an n-element array?

 (A) $n^2 \log n$
 (B) $\frac{n(n+1)}{4}$
 (C) $\frac{(n-1)(n+2)}{4}$
 (D) $\frac{n(n-1)}{4} + \sum_{i=1}^{n-1} \frac{i}{i+1}$

16. Same as above.

17. Let C be a class of sorting algorithms that remove at most one inversion after each comparison. Which of the following is a lower bound on the average number of comparisons of keys that every algorithm on C must perform while sorting an n-element array?

 (A) $n^2 \log n$
 (B) $\frac{n(n+1)}{4}$
 (C) $\frac{(n-1)(n+2)}{4}$
 (D) $\frac{n(n-1)}{4} + \sum_{i=1}^{n-1} \frac{i}{i+1}$

18. Same as above.

19. What is the shortest *ipl* (the *internal path length*) in any binary tree on n nodes?

 (A) $ipl = \sum_{i=1}^{n} \lfloor \lg i \rfloor$
 (B) $ipl = (n+1)\lfloor \lg n \rfloor - 2^{\lfloor \lg n \rfloor + 1} + 2$
 (C) $ipl = (n+1)(\lg(n+1) + \varepsilon(n+1)) - 2n$, where $\varepsilon(n) = 1 + \theta - 2^{\theta}$ and $\theta = \lceil \lg n \rceil - \lg n$.
 (D) All the above.

20. Same as above.

21. How many comparisons of keys are performed in the worst case during the execution of `Mergesort` on an n-element array?

    (A) $n(\lceil \log_2 n \rceil + 1) - 2^{\lceil \log_2 n \rceil}$
    (B) $n(\lfloor \log_2 (n+1) \rfloor + 1) - 2^{\lfloor \log_2 (n+1) \rfloor}$
    (C) $n\lceil \log_2 (n+1) \rceil - 2^{\lceil \log_2 (n+1) \rceil} + 1$
    (D) $n\lfloor \log_2 n \rfloor - 2^{\lfloor \log_2 n \rfloor} + 1$

22. Same as above.

PART II: ESSAY-TYPE QUESTIONS

For graduate students, only the questions marked as "Graduate Credit" will earn credits.

23. Prove by induction the following equality:

$$\sum_{i=1}^{n}(2i-1) = n^2$$

24. Prove that for every integer $n \geq 1$,

$$\lfloor \log_2 n \rfloor + 1 = \lceil \log_2 (n+1) \rceil.$$

25. Find a best-case array E with 15 distinct elements for Quicksort. How many comparisons did QuickSort made while sorting that array?

26. **Graduate Credit** Prove that for any integer $b, c \geq 2$, if

$$f(n) \in \Omega(n^{\log_c b + \epsilon})$$

for some positive ϵ, and

$$f(n) \in O(n^{\log_c b + \delta})$$

for some $\delta > \epsilon$, then any solution of the recurrence equation

$$T(n) = bT(\frac{n}{c}) + f(n)$$

satisfies

$$T(n) \in \Theta(f(n)).$$

THAT'S ALL, FOLKS.