A brief summary of computation of $epl(T_n)$ in balanced 2-tree T_n

Note Title 3/3/2012

Note: This summary is not supposed to substitute the 2-tree; it just helps studying it.

Critical definitions that you need to memorize:

- def. of 2-tree (including $\underbrace{\text{external}}_{n+1}$ & $\underbrace{\text{internal nodes}}_{n}$)
- def. of level in 2-tree
- def. of depth in a 2-tree
- number of decisions made along a path in a 2-tree
- def. of external path length
- def. of internal path length

- def. of balanced 2-tree (one that has minimal epl)

Basic properties of 2-trees that you need to memorize

- n internal nodes $\Rightarrow$ $n+1$ external nodes
- level of child = 1 + level of its parent
- level of root = 0
- $epl(T_n) = ipl(T_n) + 2n$ (n = number of internal nodes in T_n)

- A 2-tree is balanced iff it does not have external nodes above the last level (defined as the last level containing any internal nodes)
- The level of k-th consecutive node (in level-by-level enumeration from 1 on) is $\lfloor \lg k \rfloor$ in a balanced 2-tree.
- The depth of a balanced 2-tree with n internal nodes is $\lfloor \lg n \rfloor$

- The depth of a balanced 2-tree with m external nodes is $\lceil \lg m \rceil$.
- Each balanced 2-tree with n internal nodes has a minimal depth of all 2-trees with n internal nodes.
- Each balanced 2-tree with m external nodes has a minimal depth of all 2-trees with m external nodes.

Equation to derive easily epl(T_n) in a balanced 2-tree with n nodes:

$$\begin{cases} x + \frac{y}{2} = 2^{\lfloor \lg n \rfloor} \\ x + y = n+1 \end{cases}$$

Where x is the number of external nodes in the last level of T_n, and y is the number of external nodes below that level.

$$\mathrm{epl}(T_n) = x \cdot \lfloor \lg n \rfloor + y \cdot (\lfloor \lg n \rfloor + 1)$$

$\mathrm{epl}(T)$ in a balanced 2-tree with m nodes is

$$m(\lceil \lg m \rceil + 1) - 2^{\lceil \lg m \rceil}$$

If you want to get A you need to be able to prove things. In particular, the following proofs are considered fundamental:

- proof that a balanced 2-tree with n nodes has depth $\lfloor \lg n \rfloor$
  (it uses definition of floor:
  $$\lfloor x \rfloor = \max\{n \in \mathbb{N} \mid n \leq x\})$$
- proof of $epl(T) = ipl(T) + 2n$
  (by induction)
- proof that a 2-tree is balanced iff it has no external nodes above

the last level (proof by contradiction)

- derivation of the formula for $epl(T_n)$ in a balanced 2-tree T_n with n nodes (this one is more difficult than others)