

A Clever Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Marek A. Suchenek ©

Department of Computer Science
CSU Dominguez Hills



November 14, 2015

Copyright by Dr. Marek A. Suchenek.
This material is intended for future publication.
Absolutely positively no copying no printing
no sharing no distributing of ANY kind please.

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact

$$\sum_{i=2}^{N-1} \lfloor \lg i \rfloor = N \lfloor \lg(N-1) \rfloor - 2^{\lfloor \lg(N-1) \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

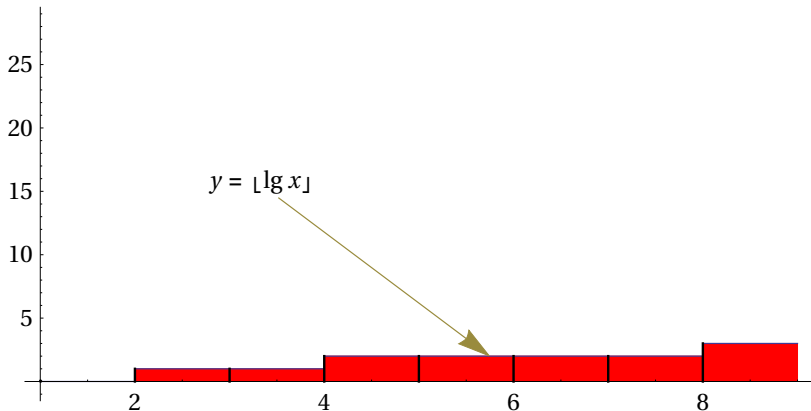


Figure : $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor = \int_{x=1}^N \lfloor \lg x \rfloor dx$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

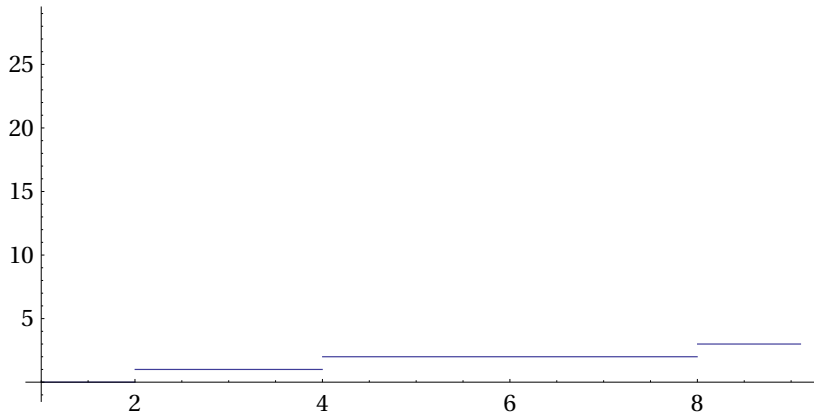


Figure : Compute $\int_{i=1}^N \lfloor \lg x \rfloor dx$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

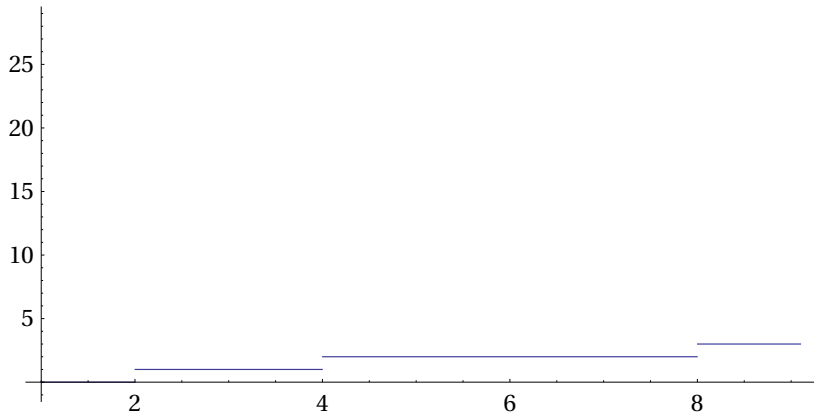


Figure : Try $N \lfloor \lg N \rfloor$ as $\int_{x=1}^N \lfloor \lg x \rfloor dx$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

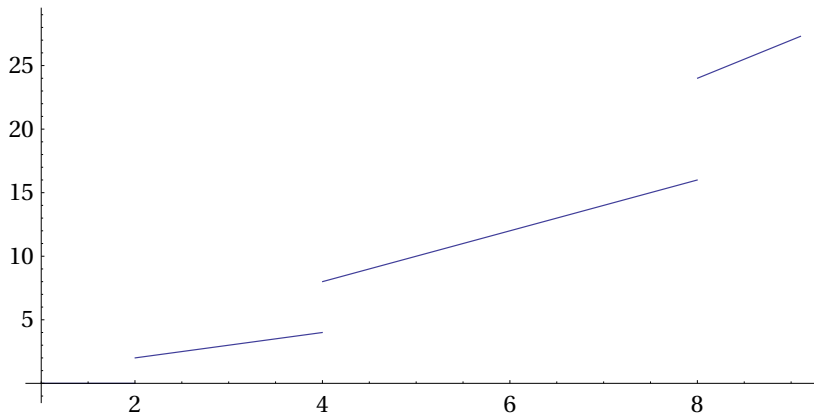


Figure : Try $N \lfloor \lg N \rfloor$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

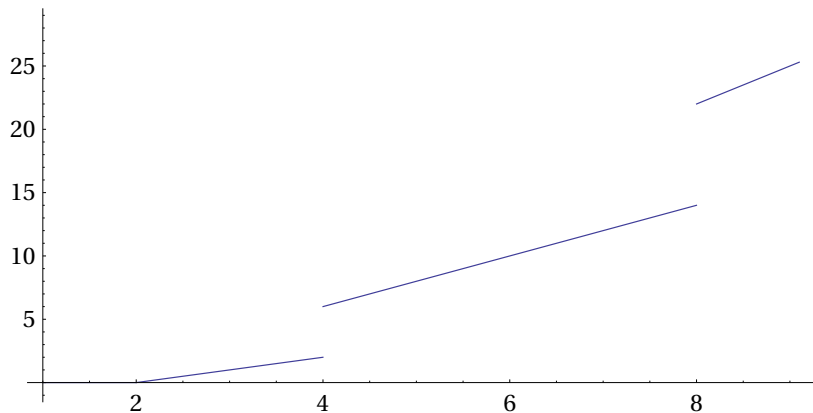


Figure : Try $N \lfloor \lg N \rfloor - 2^1$ ($N \geq 2$)

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

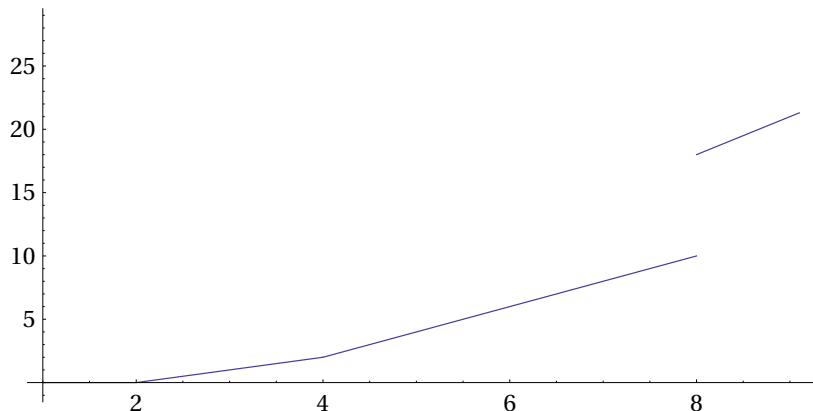


Figure : Try $N \lfloor \lg N \rfloor - 2^1$ ($N \geq 2$) -2^2 ($N \geq 4$)

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

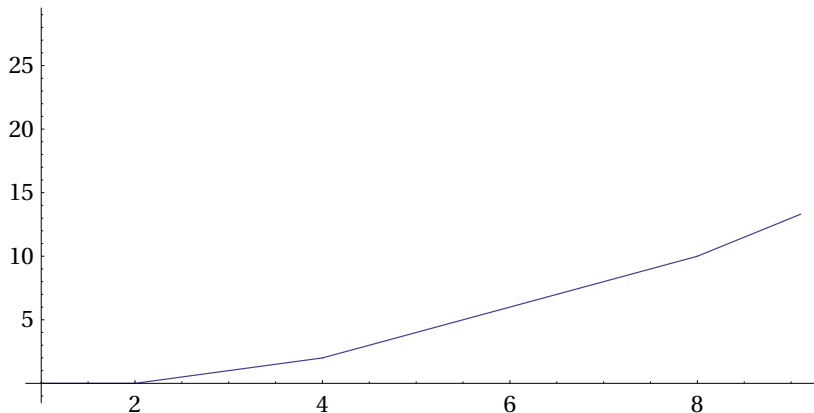


Figure : Try $N \lfloor \lg N \rfloor - 2^1 (N \geq 2) - 2^2 (N \geq 4) - 2^3 (N \geq 8)$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

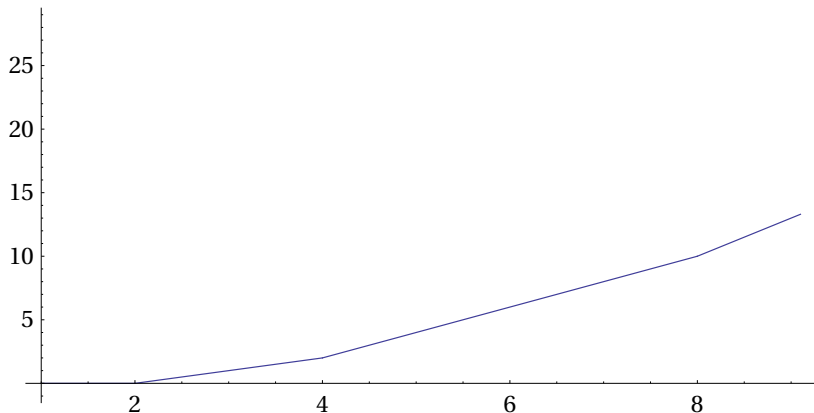


Figure : Try $N \lfloor \lg N \rfloor - (2^1 + 2^2 + \dots + 2^{\lfloor \lg N \rfloor})$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

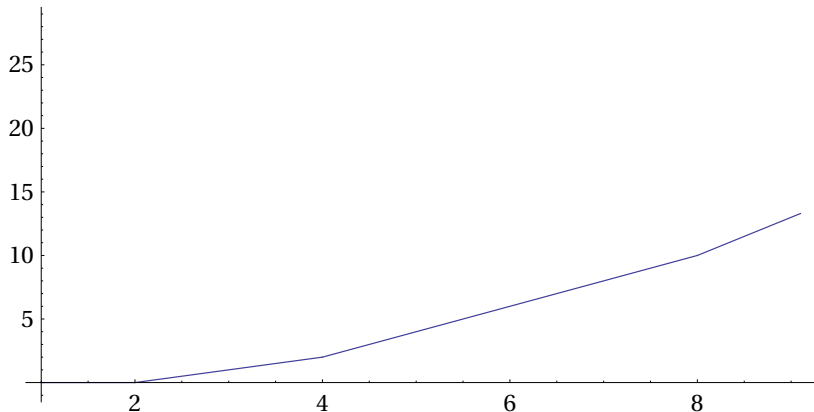


Figure : Try $N \lfloor \lg N \rfloor - \sum_{i=1}^{\lfloor \lg N \rfloor} 2^i$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

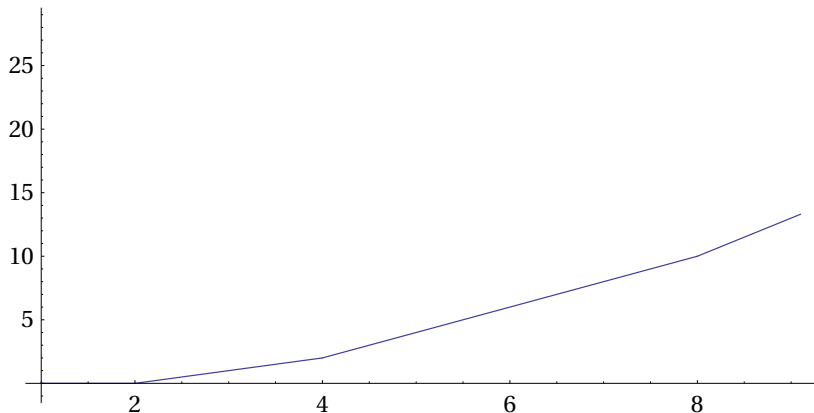


Figure : Try $N \lfloor \lg N \rfloor - 2 \lfloor \lg N \rfloor + 1 + 2$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

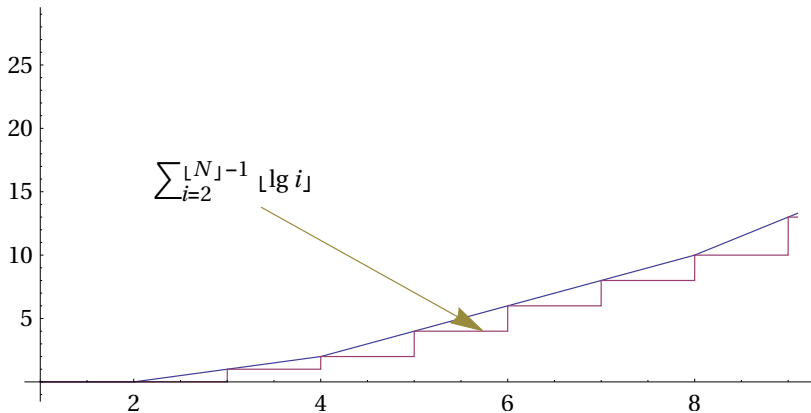


Figure : $N \lfloor \lg N \rfloor - 2^{\lfloor \lg N \rfloor + 1} + 2 = \sum_{i=2}^{\lfloor N \rfloor - 1} \lfloor \lg i \rfloor$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

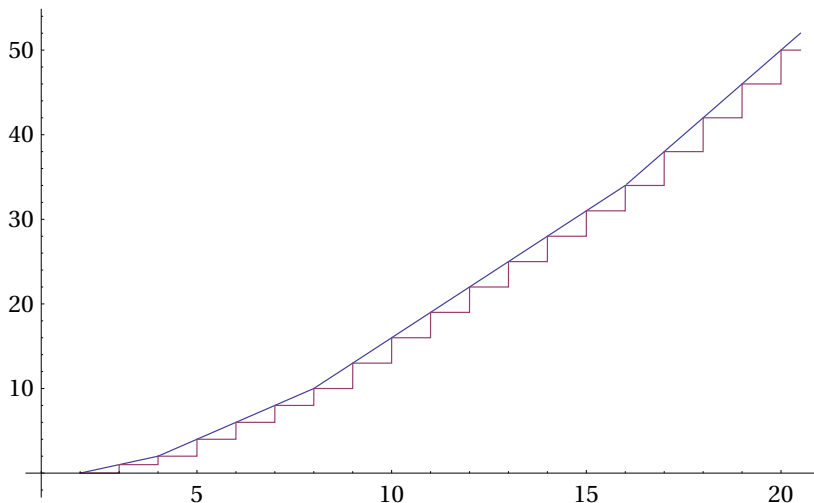


Figure : $N \lfloor \lg N \rfloor - 2^{\lfloor \lg N \rfloor + 1} + 2 = \sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - continued

$$\sum_{i=2}^{N-1} \lfloor \lg i \rfloor = N \lfloor \lg N \rfloor - 2^{\lfloor \lg N \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - continued

$$\sum_{i=2}^{n+1-1} \lfloor \lg i \rfloor = (n+1) \lfloor \lg (n+1) \rfloor - 2^{\lfloor \lg (n+1) \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Another useful formula for $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$.

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - continued

$$\sum_{i=2}^{N-2} \lfloor \lg i \rfloor = (N-1) \lfloor \lg(N-1) \rfloor - 2^{\lfloor \lg(N-1) \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - continued

$$\sum_{i=2}^{N-1} \lfloor \lg i \rfloor = (N-1) \lfloor \lg(N-1) \rfloor - 2^{\lfloor \lg(N-1) \rfloor + 1} + 2 + \lfloor \lg(N-1) \rfloor.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - Bingo!

$$\sum_{i=2}^{N-1} \lfloor \lg i \rfloor = N \lfloor \lg(N-1) \rfloor - 2^{\lfloor \lg(N-1) \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - Bingo!

$$\sum_{i=2}^{n+1-1} \lfloor \lg i \rfloor = (n+1) \lfloor \lg(n+1-1) \rfloor - 2^{\lfloor \lg(n+1-1) \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - Bingo!

$$\sum_{i=2}^n \lfloor \lg i \rfloor = (n+1) \lfloor \lg n \rfloor - 2^{\lfloor \lg n \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

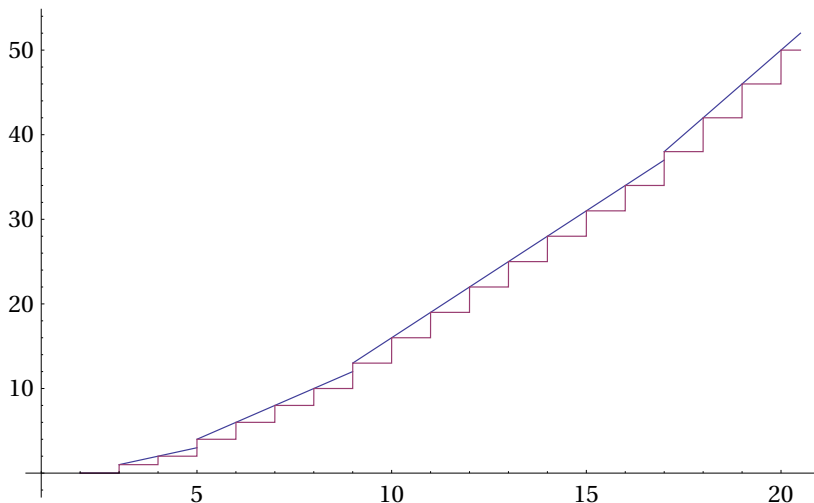


Figure : $N \lfloor \lg(N-1) \rfloor - 2 \lfloor \lg(N-1) \rfloor + 1 + 2$ and $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

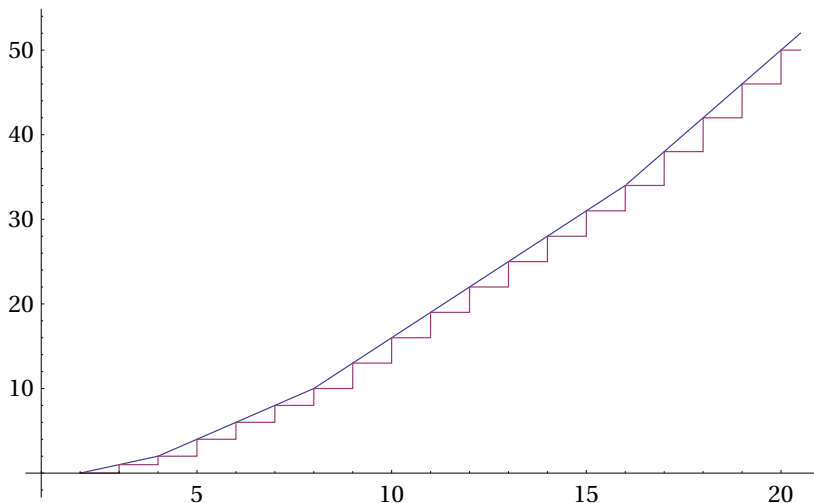


Figure : $N \lfloor \lg N \rfloor - 2^{\lfloor \lg N \rfloor + 1} + 2$ and $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

DONE!