A Clever Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

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November 14, 2015

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact

$$\sum_{i=2}^{N-1} \lfloor \lg i \rfloor = N\lfloor \lg(N-1) \rfloor - 2^{\lfloor \lg(N-1) \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

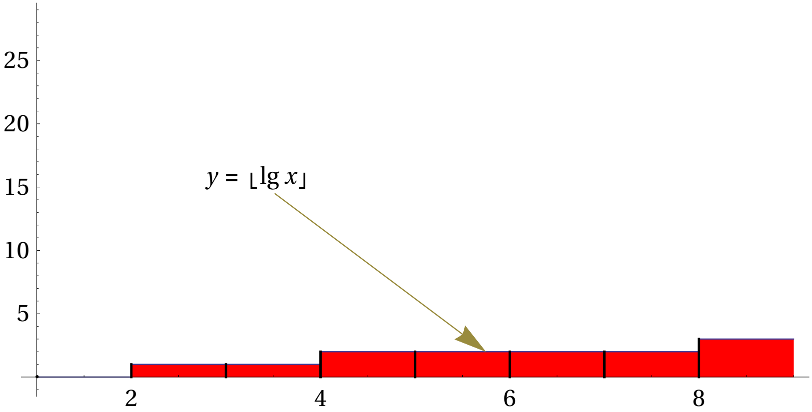

Figure : $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor = \int_{x=1}^{N} \lfloor \lg x \rfloor dx$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

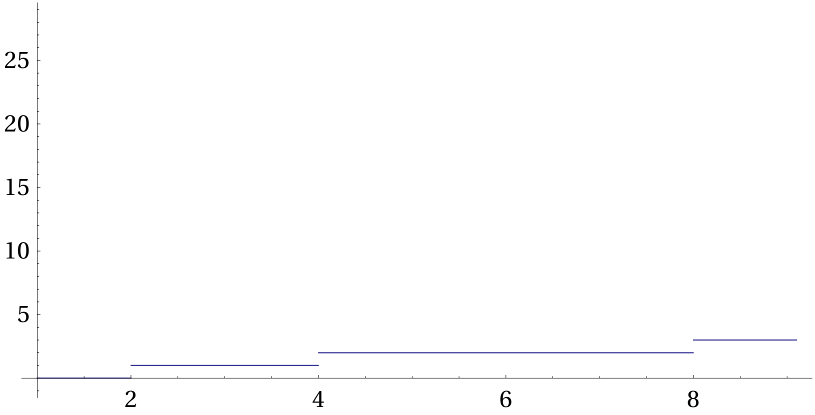

Figure : Compute $\int_{i=1}^{N} \lfloor \lg x \rfloor dx$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

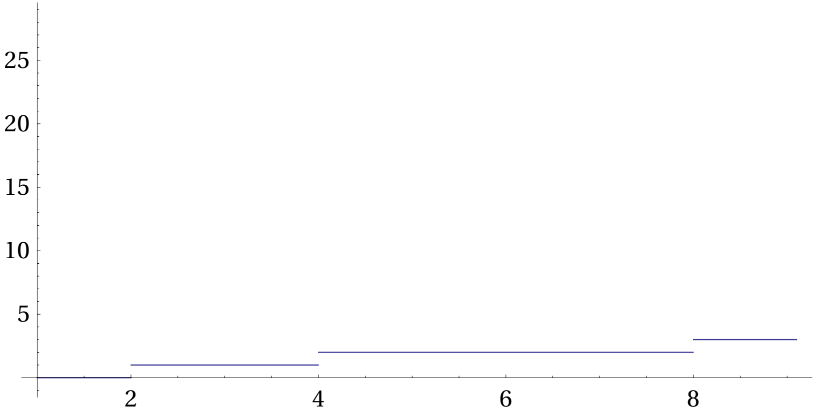

Figure : Try $N \lfloor \lg N \rfloor$ as $\int_{x=1}^{N} \lfloor \lg x \rfloor dx$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

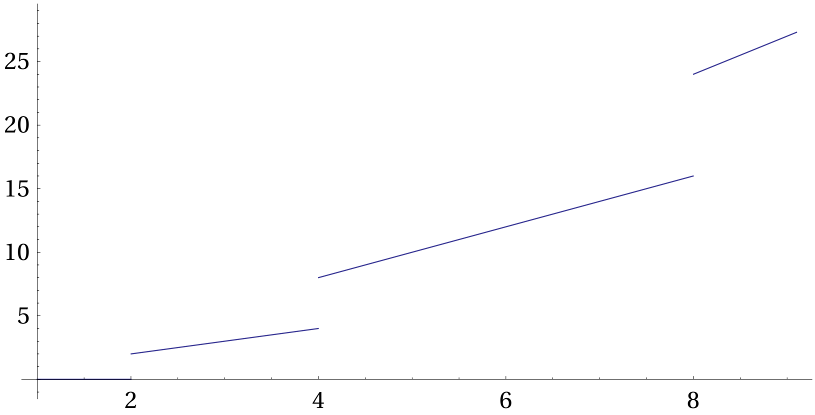

Figure : Try $N\lfloor \lg N \rfloor$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

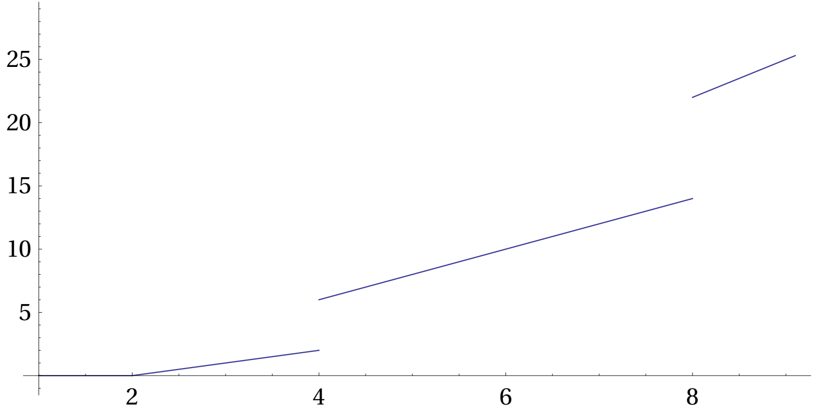

Figure : Try $N\lfloor \lg N \rfloor - 2^1$ $(N \geq 2)$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

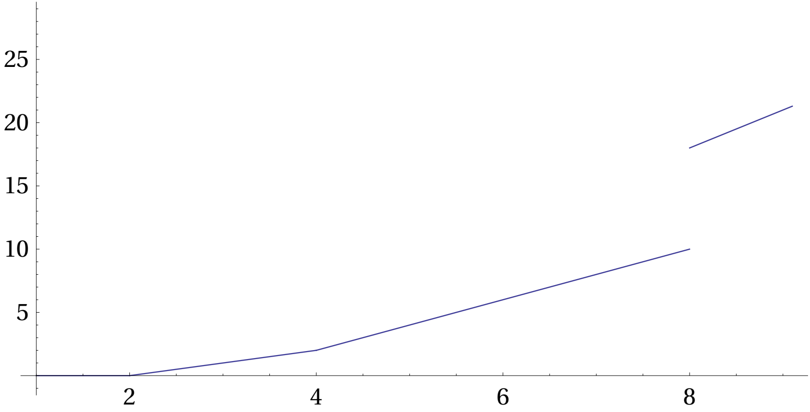

Figure : Try $N\lfloor \lg N \rfloor - 2^1$ $(N \geq 2)$ -2^2 $(N \geq 4)$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

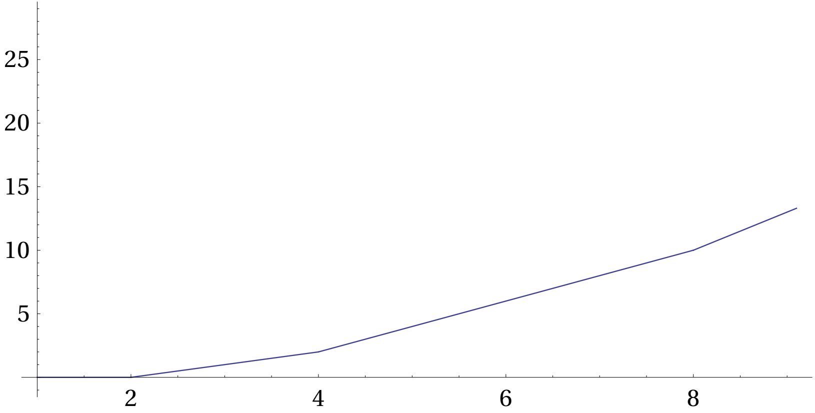

Figure : Try $N\lfloor \lg N \rfloor - 2^1$ $(N \geq 2)$ -2^2 $(N \geq 4)$ -2^3 $(N \geq 8)$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

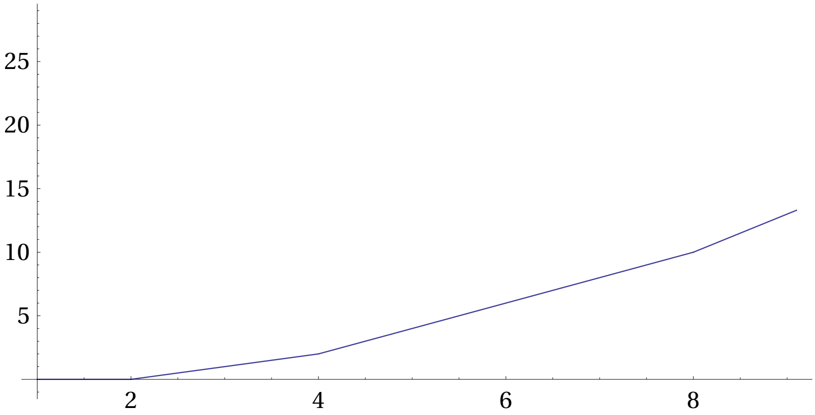

Figure : Try $N\lfloor \lg N \rfloor - (2^1 + 2^2 + \ldots + 2^{\lfloor \lg N \rfloor})$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

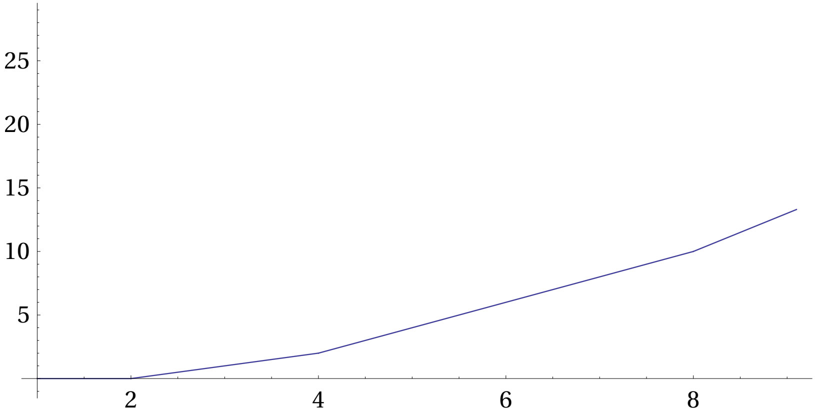

Figure : Try $N\lfloor \lg N \rfloor - \sum_{i=1}^{\lfloor \lg N \rfloor} 2^i$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

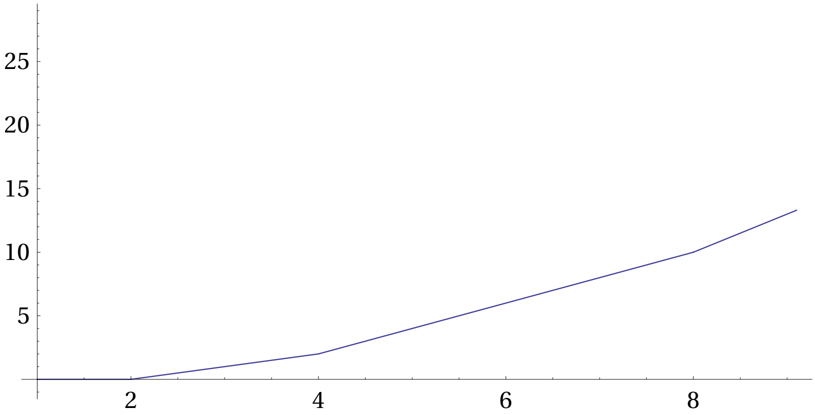

Figure : Try $N\lfloor \lg N \rfloor - 2^{\lfloor \lg N \rfloor + 1} + 2$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

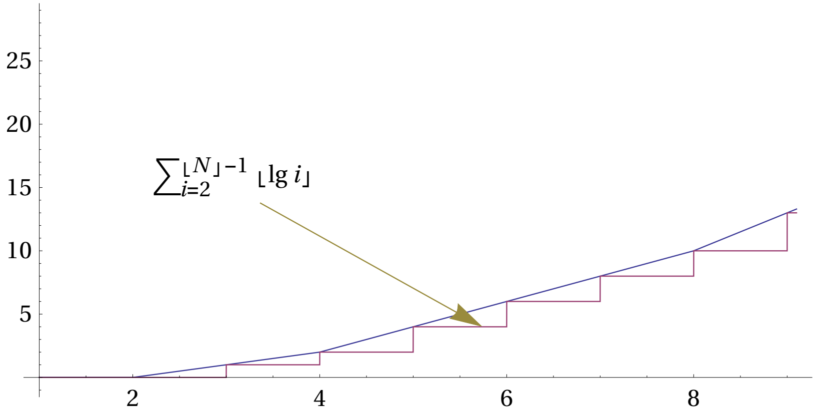

Figure : $N\lfloor \lg N \rfloor - 2^{\lfloor \lg N \rfloor + 1} + 2 = \sum_{i=2}^{\lfloor N \rfloor -1} \lfloor \lg i \rfloor$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

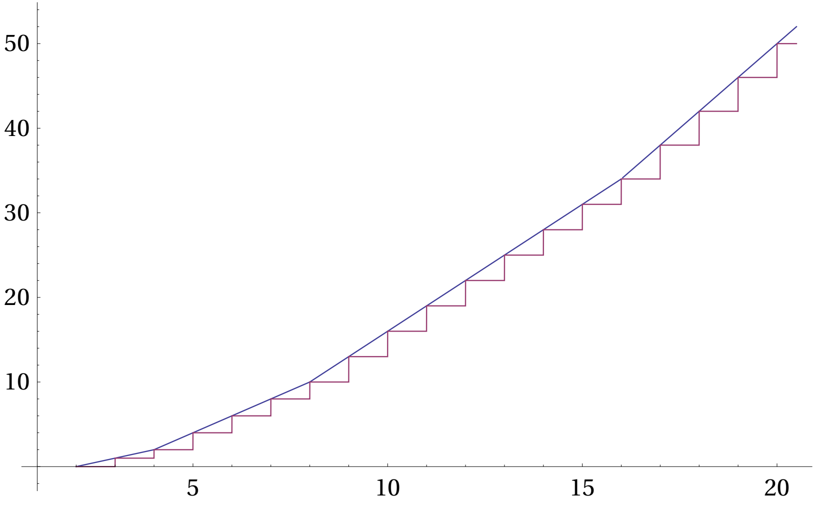

Figure : $N\lfloor \lg N \rfloor - 2^{\lfloor \lg N \rfloor + 1} + 2 = \sum_{i=2}^{\lfloor N \rfloor - 1} \lfloor \lg i \rfloor$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - continued

$$\sum_{i=2}^{N-1} \lfloor \lg i \rfloor = N \lfloor \lg N \rfloor - 2^{\lfloor \lg N \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - continued

$$\sum_{i=2}^{n+1-1} \lfloor \lg i \rfloor = (n+1)\lfloor \lg (n+1) \rfloor - 2^{\lfloor \lg (n+1) \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Another useful formula for $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$.

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - continued

$$\sum_{i=2}^{N-2} \lfloor \lg i \rfloor = (N-1)\lfloor \lg(N-1) \rfloor - 2^{\lfloor \lg(N-1) \rfloor + 1} + 2.$$

Fact - continued

$$\sum_{i=2}^{N-1} \lfloor \lg i \rfloor = (N-1)\lfloor \lg(N-1) \rfloor - 2^{\lfloor \lg(N-1) \rfloor + 1} + 2 + \lfloor \lg(N-1) \rfloor.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - Bingo!

$$\sum_{i=2}^{N-1} \lfloor \lg i \rfloor = N \lfloor \lg(N-1) \rfloor - 2^{\lfloor \lg(N-1) \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - Bingo!

$$\sum_{i=2}^{n+1-1} \lfloor \lg i \rfloor = (n+1)\lfloor \lg(n+1-1) \rfloor - 2^{\lfloor \lg(n+1-1) \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

Fact - Bingo!

$$\sum_{i=2}^{n} \lfloor \lg i \rfloor = (n+1)\lfloor \lg n \rfloor - 2^{\lfloor \lg n \rfloor + 1} + 2.$$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

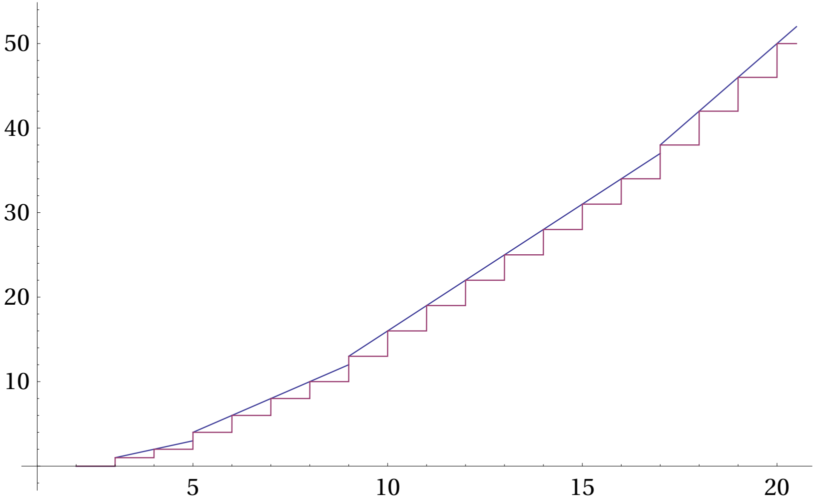

Figure : $N\lfloor \lg(N-1)\rfloor - 2^{\lfloor \lg(N-1)\rfloor+1} + 2$ and $\sum_{i=2}^{\lfloor N\rfloor -1} \lfloor \lg i \rfloor$

Computation of $\sum_{i=2}^{N-1} \lfloor \lg i \rfloor$

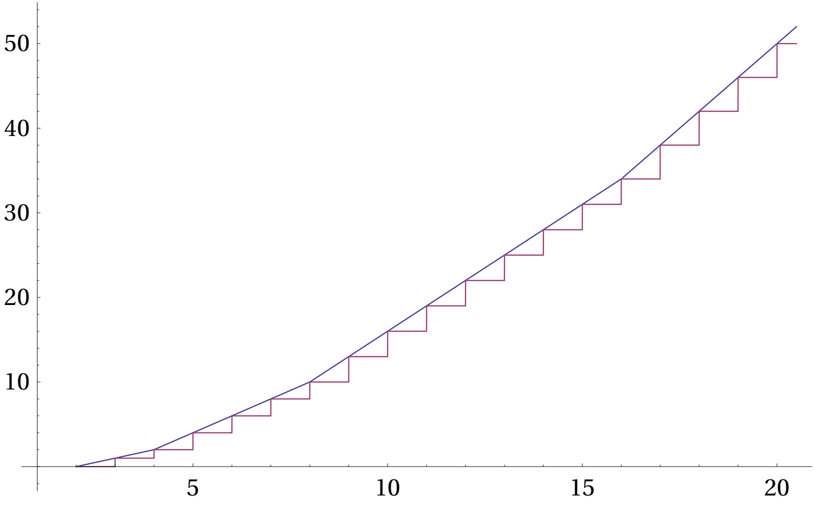

Figure : $N\lfloor \lg N \rfloor - 2^{\lfloor \lg N \rfloor + 1} + 2$ and $\sum_{i=2}^{\lfloor N \rfloor - 1} \lfloor \lg i \rfloor$

DONE!