Computer Algorithms
Introduction to Design and Analysis

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TECH

Computer Science

Analysis Tool: Probability

- Elementary events (outcomes)

Suppose that in a given situation an event, or experiment, may have any one, and only one, of k mutually exclusive outcomes, e_1, e_2, …, e_k.

- Universal set (the *universe*)
  The set of all elementary events is called the universal set and is denoted $U = \{e_1, e_2, \ldots, e_k\}$.
- Probability Pr of e_i is a function from U into reals such that:
- $0 \leq Pr(e_i) \leq 1$ for $1 \leq i \leq k$;
- $Pr(e_1) + Pr(e_2) + \ldots + Pr(e_k) = 1$

Event

- Let $S \subseteq U$. Then S is called an *event*, and
- $Pr(S) = \sum_{e \in S} Pr(e)$
- Sure event $U = \{e_1, e_2, \ldots, e_k\}$, $Pr(U) = 1$
- Impossible event 0, $Pr(0) = 0$

Complement event “not S” or –S: U – S,

$Pr(-S) = 1 - Pr(S)$

Note: Elementary event e *is* ***not*** *an event.*

{e} is.

(Events are sets of elementary events.)

Conditional Probability

- The conditional probability of an event S *given* an event T is defined as

$$\Pr(S \mid T) = \Pr(S \cap T) / \Pr(T)$$
$$= (\textstyle\sum_{e \in S \cap T} \Pr(e)) / (\textstyle\sum_{e \in T} \Pr(e))$$

- *Independent events*

S and T are stochastically independent events (or *independent events*) iff

$$\mathbf{\Pr(S \cap T) = \Pr(S) \times \Pr(T)}$$

Random variable and their Expected value

- A *random variable f* is a function from U into reals.
  - **f(e) represents an outcome of event e.**
- Expected value of a random variable f
  - **Let f be a random variable defined on a set of elementary events U. The expected value of f, denoted as E(f), is defined as**
- $E(f) = \sum_{e \in U} f(e) \times \Pr(e)$
  - **Some call it *weighted average* of f.**

- Conditional expected value of a random variable f
- $E(f \mid S) = \sum_{e \in U} f(e) \times \Pr(e \mid S) =$

$$= \sum_{e \in S} f(e) \times \Pr(e \mid S)$$