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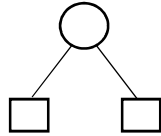
please.

Theorem For every 2-tree  $T_n$  with  $n$  internal nodes,

$$\text{epl}(T_n) = \text{ipl}(T_n) + 2n$$

Proof by induction on  $n$ .

Basic step ( $n = 1$ )



$$\text{ipl}(T_1) = 0$$

$$\text{epl}(T_1) = 1 + 1 = 2$$

$$\text{So, } \text{epl}(T_1) = \text{ipl}(T_1) + 2 \times 1$$

This completes the basic step.

Inductive step

Inductive hypothesis assume that for sum  $n \geq 1$ , and for every 2-tree  $T_n$  with  $n$  internal nodes,

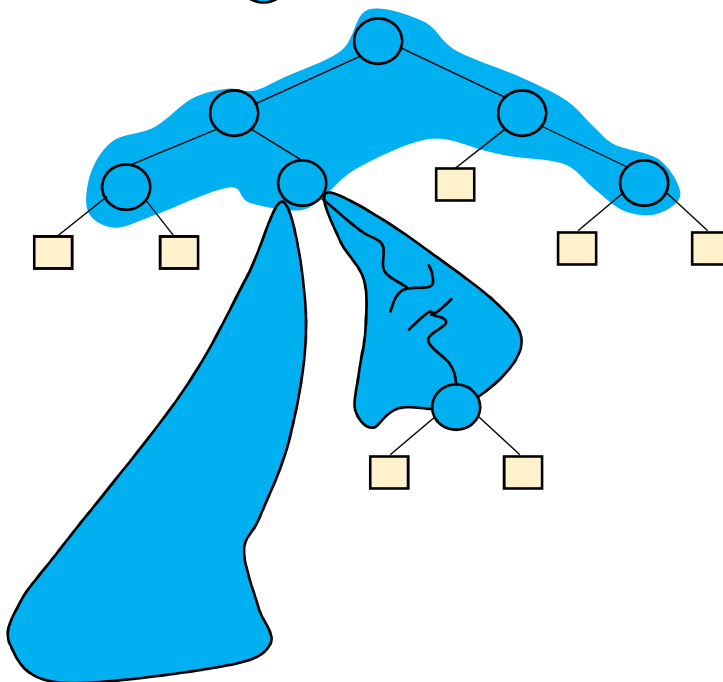
$$\text{epl}(T_n) = \text{ipl}(T_n) + 2n$$

We are going to prove that for every 2-tree  $T_{n+1}$  with  $n + 1$  internal nodes,

$$\text{epl}(T_{n+1}) = \text{ipl}(T_{n+1}) + 2(n + 1).$$

Let  $T_{n+1}$  be any 2-tree with  $n + 1$  internal nodes.

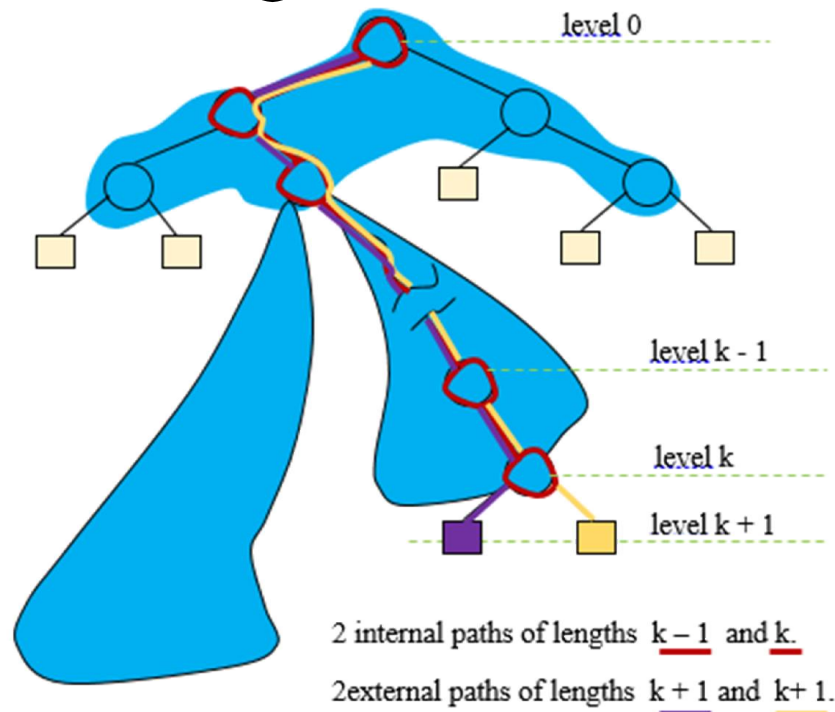
2-tree  $T_{n+1}$  with  $n + 1$  internal nodes (●)



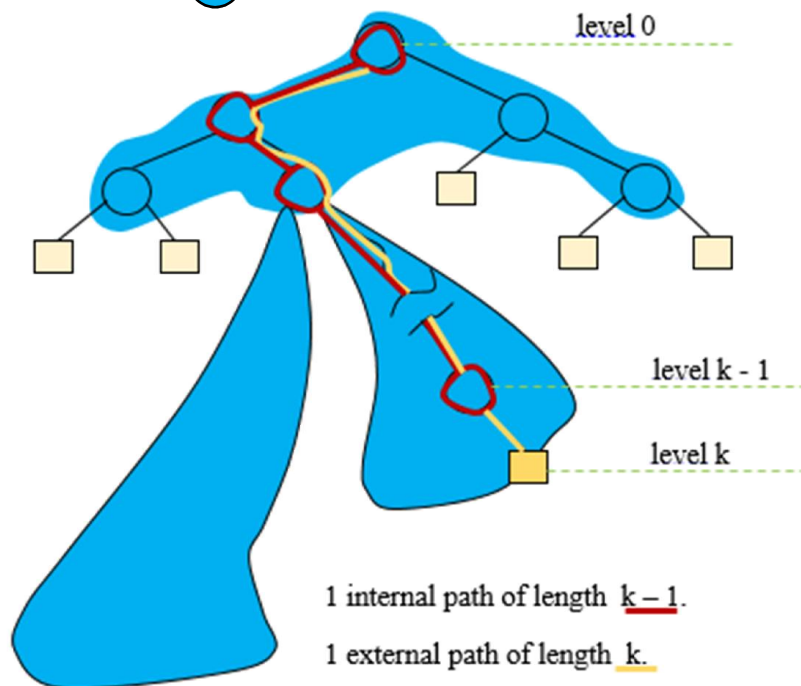
Let's remove one internal leaf from  $T_{n+1}$ . (It must have an internal leaf because it is non-empty and finite.)  
 The resulting 2-tree  $T_n$  has  $n$  internal leaves, so the inductive hypothesis does apply!

$$\text{epl}(T_n) = \text{ipl}(T_n) + 2n$$

2-tree  $T_{n+1}$  with  $n+1$  internal nodes (●)



2-tree  $T_n$  with  $n$  internal nodes (●)



The differences are:

$$\begin{aligned} & \text{epl}(T_{n+1}) - \text{epl}(T_n) \\ &= k + 1 + k + 1 - k \\ &= k + 2 \end{aligned}$$

$$\begin{aligned} & \text{ipl}(T_{n+1}) - \text{ipl}(T_n) \\ &= k - 1 + k - (k - 1) = k \end{aligned}$$

Now,

$$\begin{aligned} \text{epl}(T_{n+1}) &= \text{epl}(T_n) + k + 2 \\ &= (\text{by the inductive hypothesis}) \\ & \text{ipl}(T_n) + 2n + k + 2 \\ &= \text{ipl}(T_{n+1}) + 2(n + 1). \end{aligned}$$

This completes the inductive step which completes the proof.