CSC 401

Lectures on **Analysis of Algorithms**

by

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Computer Science
CSUDH

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CSC 401

Lecture 1
Mathematical Background

Short Review

Mathematical Background

Sets, Tuples, Relations

Mathematical Background

Sets, Tuples, Relations

Empty set

Mathematical Background

Sets, Tuples, Relations

Empty set   0

Mathematical Background

Sets, Tuples, Relations

Empty set 0 { }

Mathematical Background

Sets, Tuples, Relations

Empty set 0 { }

A non-empty set

Mathematical Background

Sets, Tuples, Relations

Empty set 0 { }

A non-empty set {a, b, c}

Mathematical Background

Sets, Tuples, Relations

Empty set 0 { }

A non-empty set {a, b, c}

{0, … , n-1}

Mathematical Background

Sets, Tuples, Relations

Empty set 0 { }

A non-empty set {a, b, c}

{0, … , n-1} the set of all natural numbers < n

Mathematical Background

$\{0, \ldots, n-1\}$ the set of all natural numbers $< n$

$\{ i \mid i \text{ is a natural number} < n \}$

Mathematical Background

{0, … , n-1}   the set of all natural numbers < n

{ i I i is a natural number < n}   n

Mathematical Background

{0, … , n-1}   the set of all natural numbers < n

{ i I i is a natural number < n}   n

N   the set of all natural numbers

Mathematical Background

{0, … , n-1} the set of all natural numbers < n

{ i I i is a natural number < n}   n

N the set of all natural numbers,

N = {0, …}

Mathematical Background

{0, … , n-1}   the set of all natural numbers < n

{ i | i is a natural number < n}   n

N   the set of all natural numbers,

N = {0, …} = {0, 1, …}

Mathematical Background

{0, … , n-1}   the set of all natural numbers < n

{ i | i is a natural number < n}   n

N   the set of all natural numbers,

N = {0, …} = {0, 1, …} = {0, 1, …, n, ...}

Mathematical Background

$N = \{0, \ldots\} = \{0, 1, \ldots\} = \{0, 1, \ldots, n, \ldots\}$

Mathematical Background

$N = \{0, \ldots\} = \{0, 1, \ldots\} = \{0, 1, \ldots, n, \ldots\}$

is an **infinite** set

Mathematical Background

$N = \{0, \ldots\} = \{0, 1, \ldots\} = \{0, 1, \ldots, n, \ldots\}$

is an **infinite** set

$i \in N$

Mathematical Background

$N = \{0, \ldots\} = \{0, 1, \ldots\} = \{0, 1, \ldots, n, \ldots\}$

is an **infinite** set

$i \in N$ means: i is a natural number

Mathematical Background

$N = \{0, \ldots\} = \{0, 1, \ldots\} = \{0, 1, \ldots, n, \ldots\}$

is an **infinite** set

$i \in N$ means: i is a natural number

Even =

Mathematical Background

$N = \{0, \ldots\} = \{0, 1, \ldots\} = \{0, 1, \ldots, n, \ldots\}$

is an **infinite** set

$i \in N$ means: i is a natural number

$\text{Even} = \{2i \mid i \in N\}$

Mathematical Background

$N = \{0, \ldots\} = \{0, 1, \ldots\} = \{0, 1, \ldots, n, \ldots\}$

is an **infinite** set

$i \in N$ means: i is a natural number

$Even = \{2i \mid i \in N\}$ Odd =

Mathematical Background

$N = \{0, \ldots\} = \{0, 1, \ldots\} = \{0, 1, \ldots, n, \ldots\}$

is an **infinite** set

$i \in N$ means: i is a natural number

$Even = \{2i \mid i \in N\}$ $Odd = \{2i + 1 \mid i \in N\}$

$Even = \{0, 2, 4, \ldots\}$ $Odd = \{1, 3, 5, \ldots\}$

Mathematical Background

Even = $\{2i \mid i \in N\}$ Odd = $\{2i + 1 \mid i \in N\}$

Even = $\{0, 2, 4, \ldots\}$ Odd = $\{1, 3, 5, \ldots\}$

Mathematical Background

Even = $\{2i \mid i \in N\}$ Odd = $\{2i + 1 \mid i \in N\}$

Even = $\{0, 2, 4, \ldots\}$ Odd = $\{1, 3, 5, \ldots\}$

Even $\subseteq$ N

Mathematical Background

Even = $\{2i \mid i \in N\}$ Odd = $\{2i + 1 \mid i \in N\}$

Even = $\{0, 2, 4, \ldots\}$ Odd = $\{1, 3, 5, \ldots\}$

Even $\subseteq$ N Odd $\subseteq$ N

Mathematical Background

$Even = \{2i \mid i \in N\}$ $Odd = \{2i + 1 \mid i \in N\}$

$Even = \{0, 2, 4, \ldots\}$ $Odd = \{1, 3, 5, \ldots\}$

$Even \subseteq N$ $Odd \subseteq N$

$A \subseteq B \equiv$

Mathematical Background

$Even = \{2i \mid i \in N\}$ $Odd = \{2i + 1 \mid i \in N\}$

$Even = \{0, 2, 4, \ldots\}$ $Odd = \{1, 3, 5, \ldots\}$

$Even \subseteq N$ $Odd \subseteq N$

$A \subseteq B \equiv (\forall i)(i \in A \Rightarrow i \in B)$

Mathematical Background

$$A \subseteq B \equiv (\forall i)(i \in A \Rightarrow i \in B)$$

Mathematical Background

$$A \subseteq B \equiv (\forall i)(i \in A \Rightarrow i \in B)$$

$$A = B \equiv (\forall i)(i \in A \Leftrightarrow i \in B)$$

Mathematical Background

$A \subseteq B \equiv (\forall i)(i \in A \Rightarrow i \in B)$

$A = B \equiv (\forall i)(i \in A \Leftrightarrow i \in B)$

$\{1, 2, 5\} = \{1, 2, 2, 5, 5, 5\}$

Mathematical Background

$$A \subseteq B \equiv (\forall i)(i \in A \Rightarrow i \in B)$$

$$A = B \equiv (\forall i)(i \in A \Leftrightarrow i \in B)$$

$$\{1, 2, 5\} = \{1, 2, 2, 5, 5, 5\} = \{5, 1, 5, 2, 2, 5, 1\}$$

Mathematical Background

$A \subseteq B \equiv (\forall i)(i \in A \Rightarrow i \in B)$

$A = B \equiv (\forall i)(i \in A \Leftrightarrow i \in B)$

$\{1, 2, 5\} = \{1, 2, 2, 5, 5, 5\} = \{5, 1, 5, 2, 2, 5, 1\}$ etc

Mathematical Background

$A \subseteq B \equiv (\forall i)(i \in A \Rightarrow i \in B)$

$A = B \equiv (\forall i)(i \in A \Leftrightarrow i \in B)$

$\{1, 2, 5\} = \{1, 2, 2, 5, 5, 5\} = \{5, 1, 5, 2, 2, 5, 1\}$ etc

$A \subseteq B$ is **not** the same as $A \in B$

Mathematical Background

$A \subseteq B$ is **not** the same as $A \in B$

$Odd \subseteq N$ but $Odd \notin N$

Mathematical Background

$A \subseteq B$ is **not** the same as $A \in B$

$Odd \subseteq N$ but $Odd \notin N$

$\{1,2\} \not\subseteq \{\{1,2\}\}$ but $\{1,2\} \in \{\{1,2\}\}$

Mathematical Background

$A \subseteq B$ is **not** the same as $A \in B$

$Odd \subseteq N$ but $Odd \notin N$

$\{1,2\} \not\subseteq \{\{1,2\}\}$ but $\{1,2\} \in \{\{1,2\}\}$

Mathematical Background

$A \subseteq B$ is **not** the same as $A \in B$

$Odd \subseteq N$ but $Odd \notin N$

$\{1,2\} \not\subseteq \{\{1,2\}\}$ but $\{1,2\} \in \{\{1,2\}\}$

$\{0\} \subseteq \{0, \{0\}\}$ and $\{0\} \in \{0, \{0\}\}$

Mathematical Background

$\{0\} \subseteq \{0, \{0\}\}$ and $\{0\} \in \{0, \{0\}\}$

Mathematical Background

$\{0\} \subseteq \{0, \{0\}\}$ and $\{0\} \in \{0, \{0\}\}$

$0 \subseteq \{0, \{0\}\}$ and $0 \in \{0, \{0\}\}$

Mathematical Background

$\{0\} \subseteq \{0, \{0\}\}$ and $\{0\} \in \{0, \{0\}\}$

$0 \subseteq \{0, \{0\}\}$ and $0 \in \{0, \{0\}\}$

$(\forall B)(0 \subseteq B)$

Mathematical Background

$\{0\} \subseteq \{0, \{0\}\}$ and $\{0\} \in \{0, \{0\}\}$

$0 \subseteq \{0, \{0\}\}$ and $0 \in \{0, \{0\}\}$

$(\forall B)(0 \subseteq B)$

U the universal class

Mathematical Background

$\{0\} \subseteq \{0, \{0\}\}$ and $\{0\} \in \{0, \{0\}\}$

$0 \subseteq \{0, \{0\}\}$ and $0 \in \{0, \{0\}\}$

$(\forall B)(0 \subseteq B)$

U the universal class

contains all sets and all elements

Mathematical Background

U the universal class

Mathematical Background

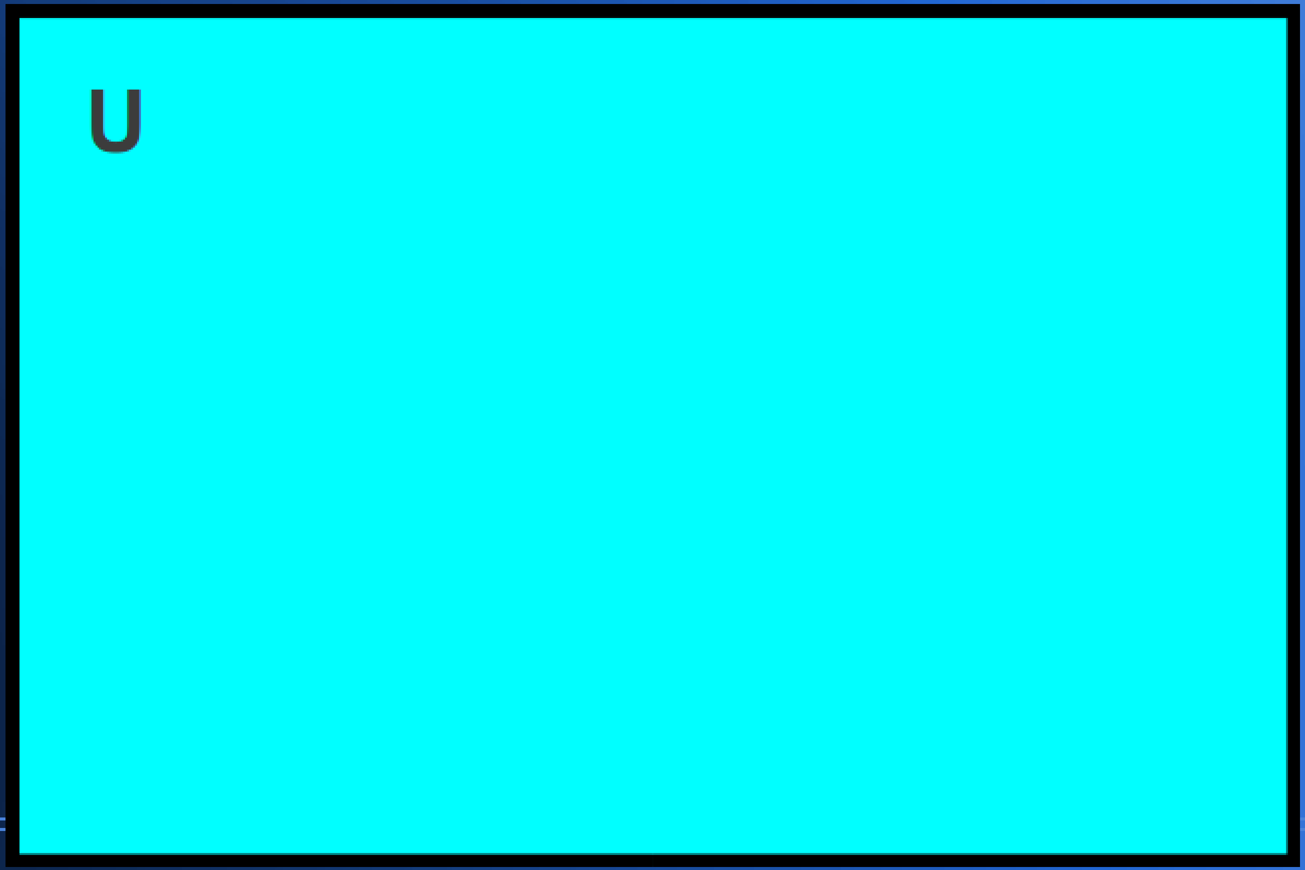
U the universal class

Algebra of sets

Mathematical Background

U the universal class

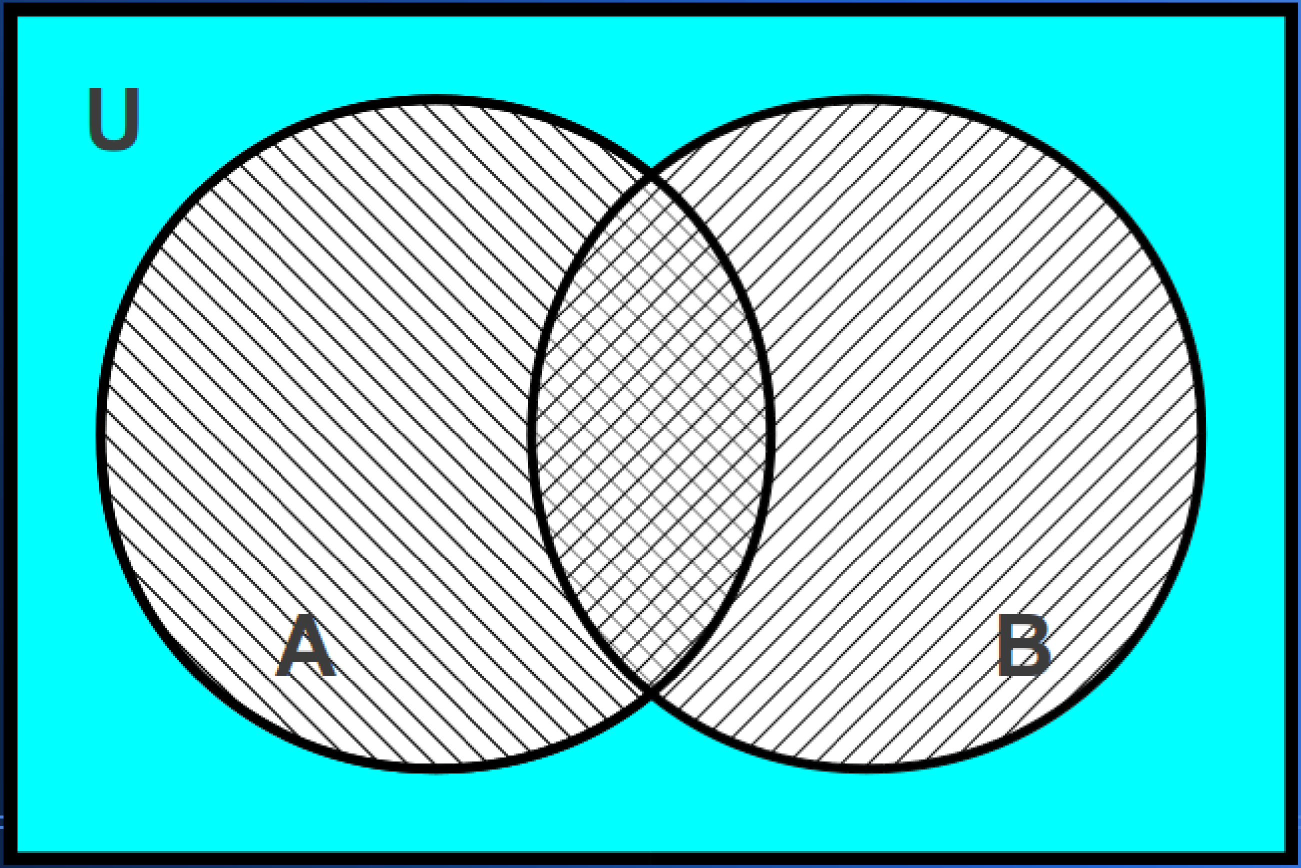
Algebra of sets



Mathematical Background

U the universal class

Algebra of sets

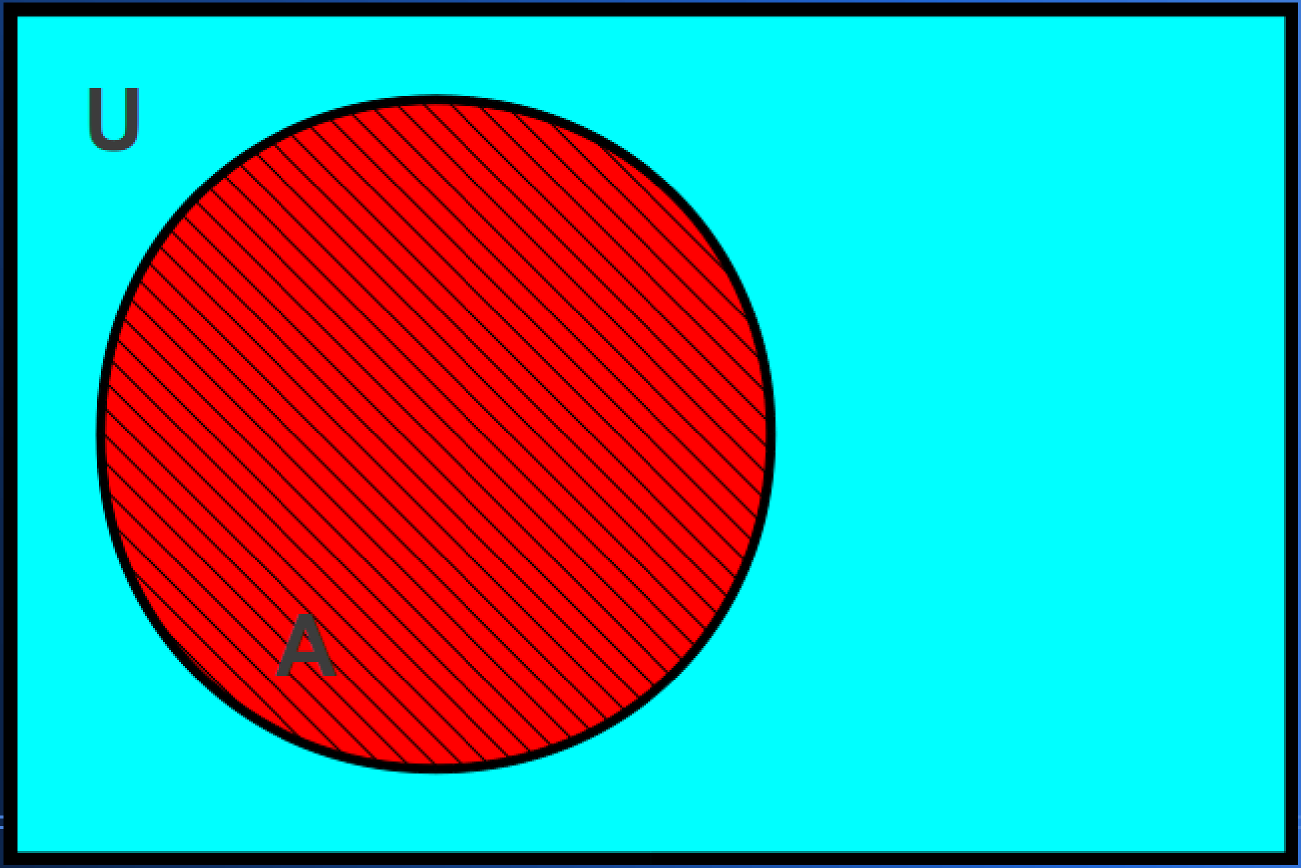


Mathematical Background

U the universal class

Algebra of sets

A (red)



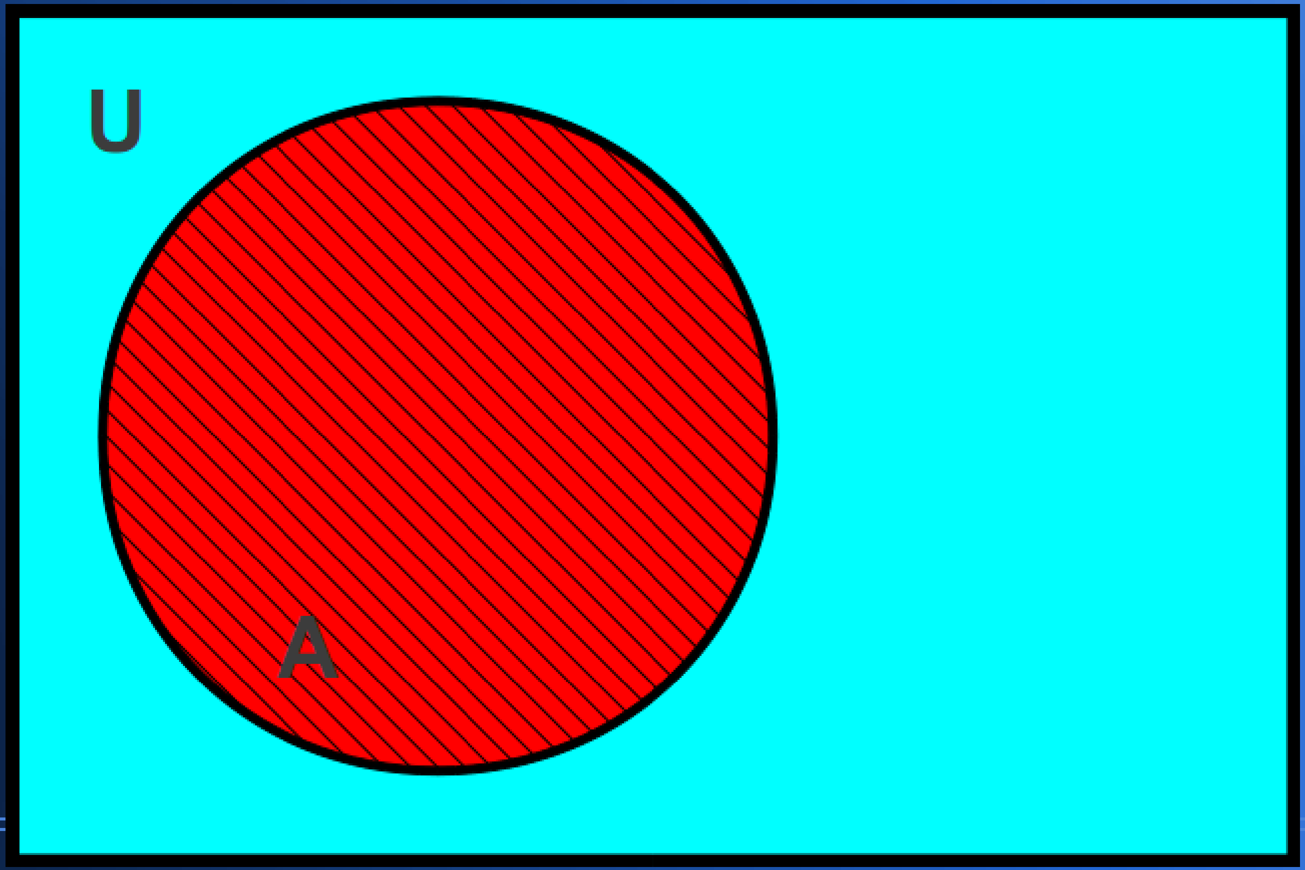
Mathematical Background

U the universal class

Algebra of sets

A (red)

-A (cyan)

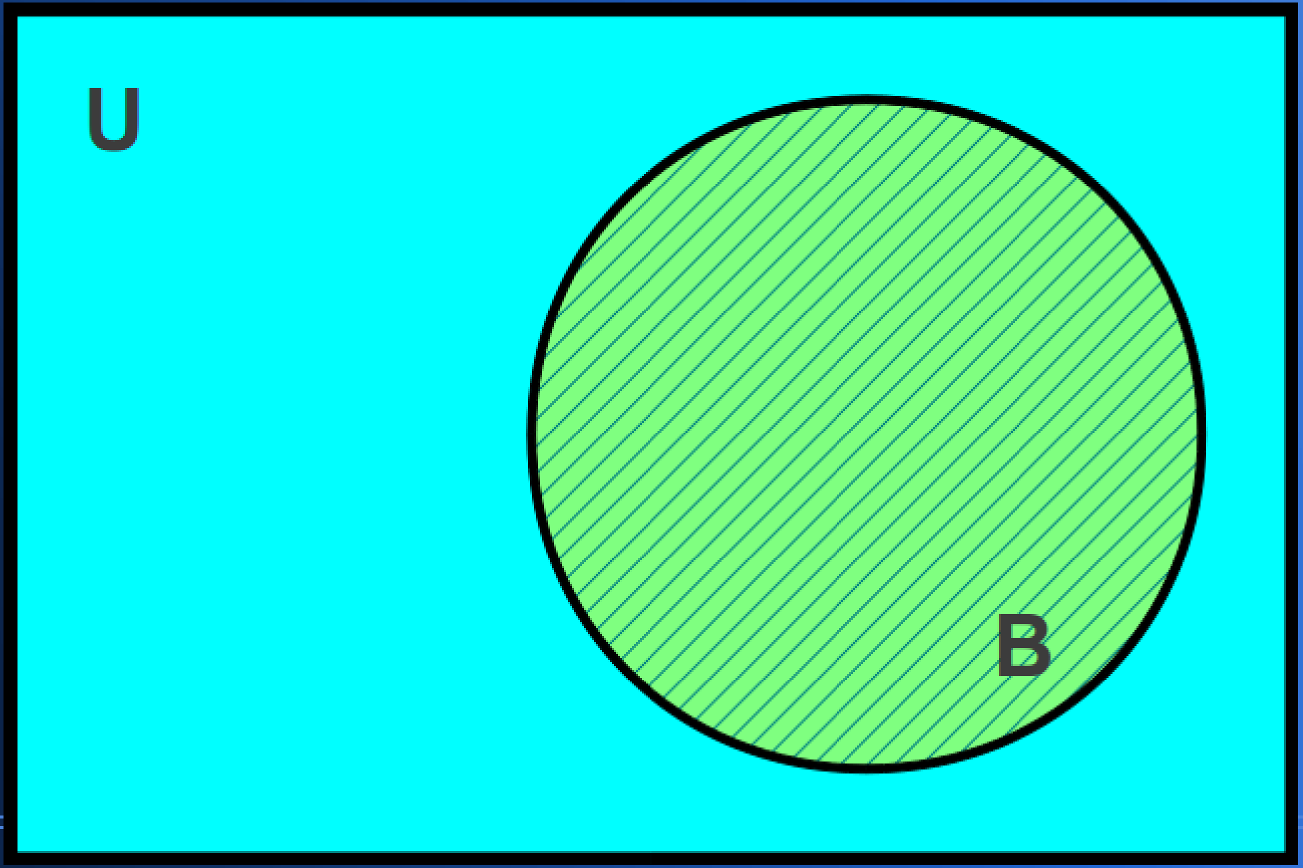


Mathematical Background

U the universal class

Algebra of sets

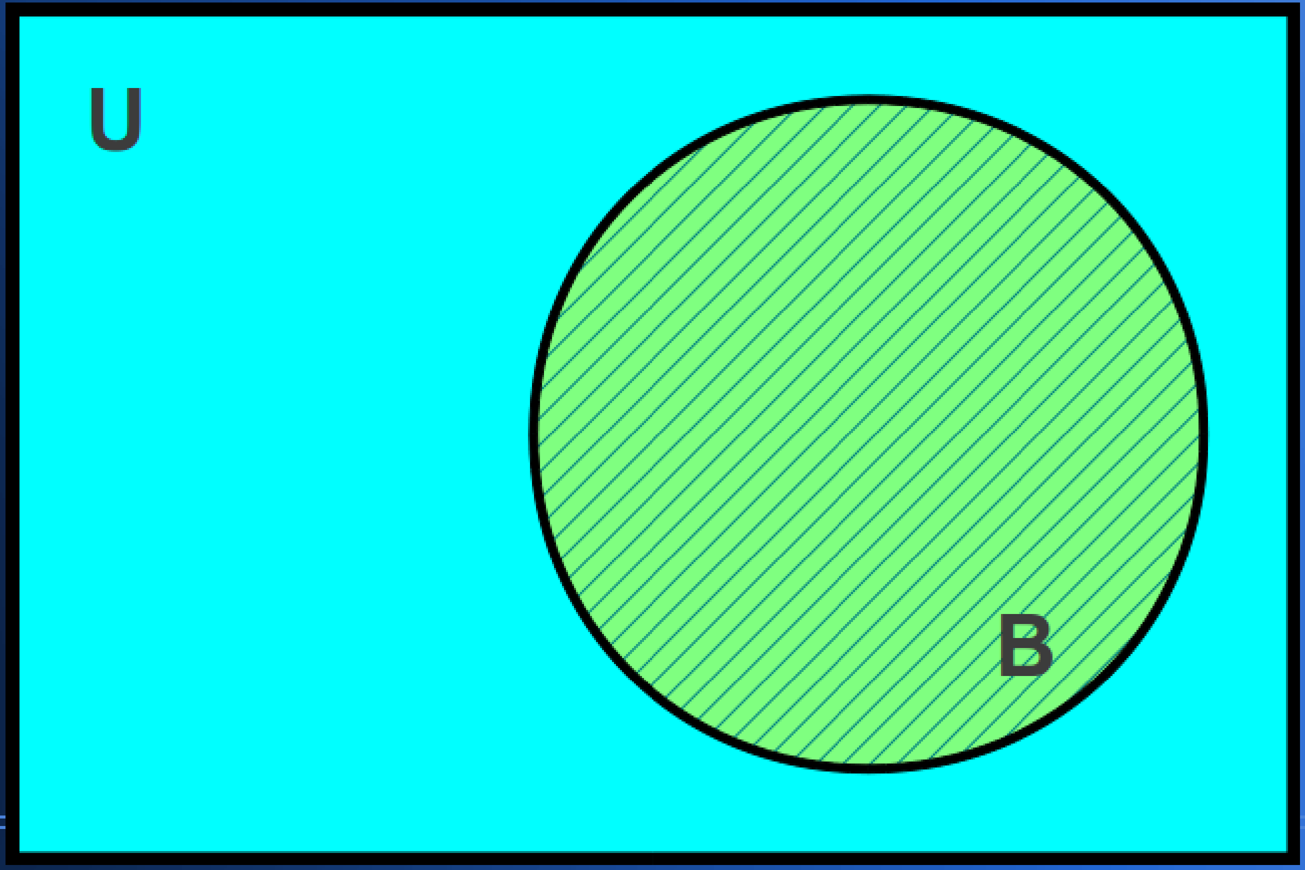
B (green)



Mathematical Background

U the universal class

Algebra of sets



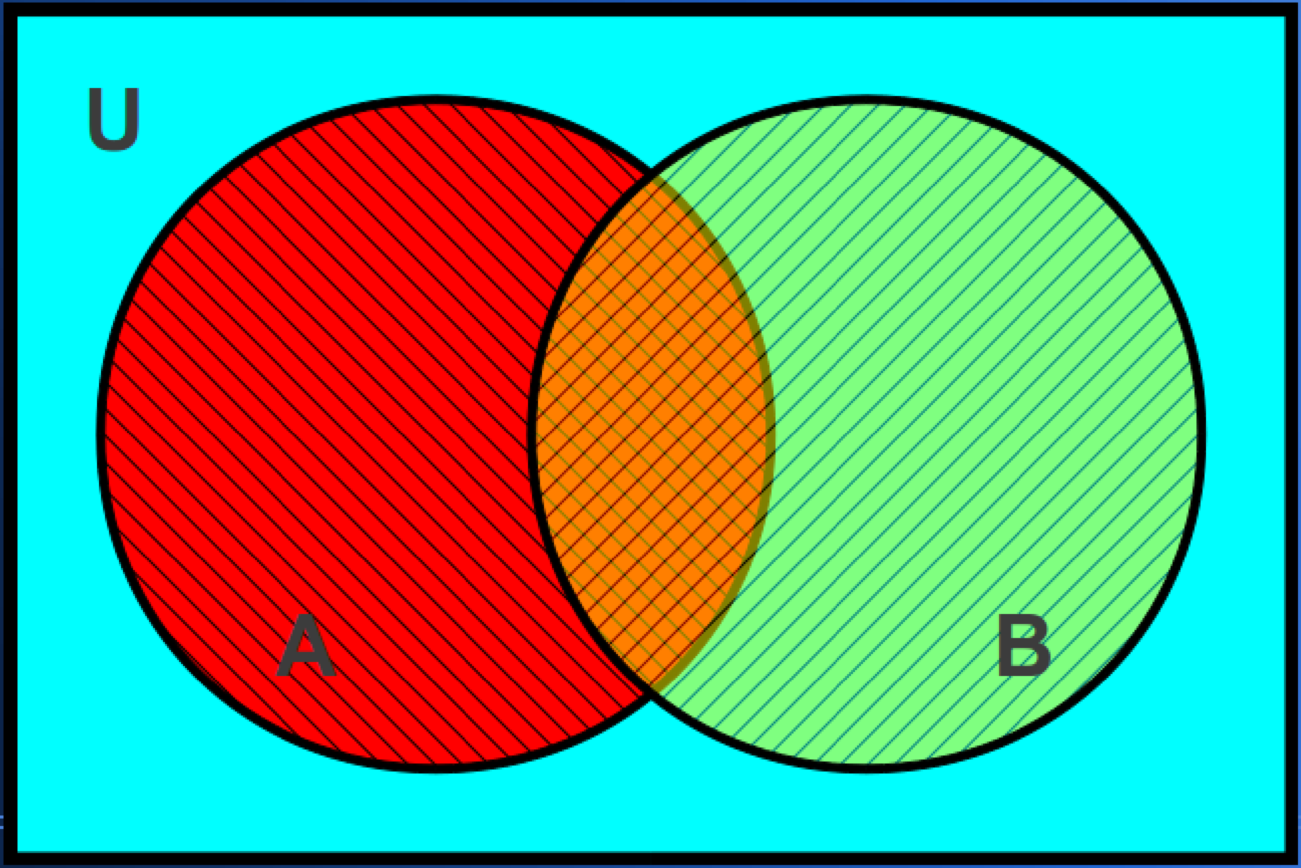
B (green)

-B (cyan)

Mathematical Background

U the universal class

Algebra of sets



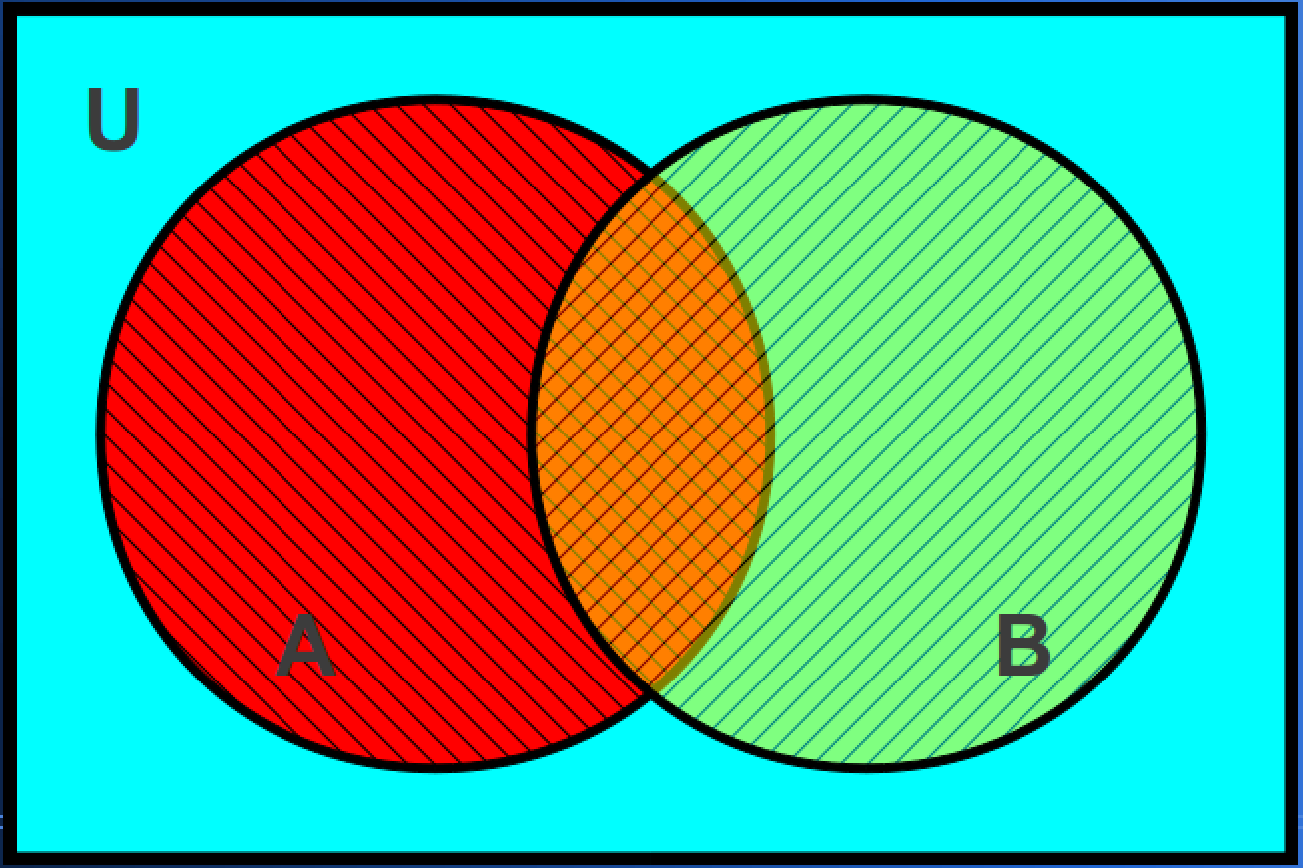
Mathematical Background

U the universal class

Algebra of sets

$A \cap B$

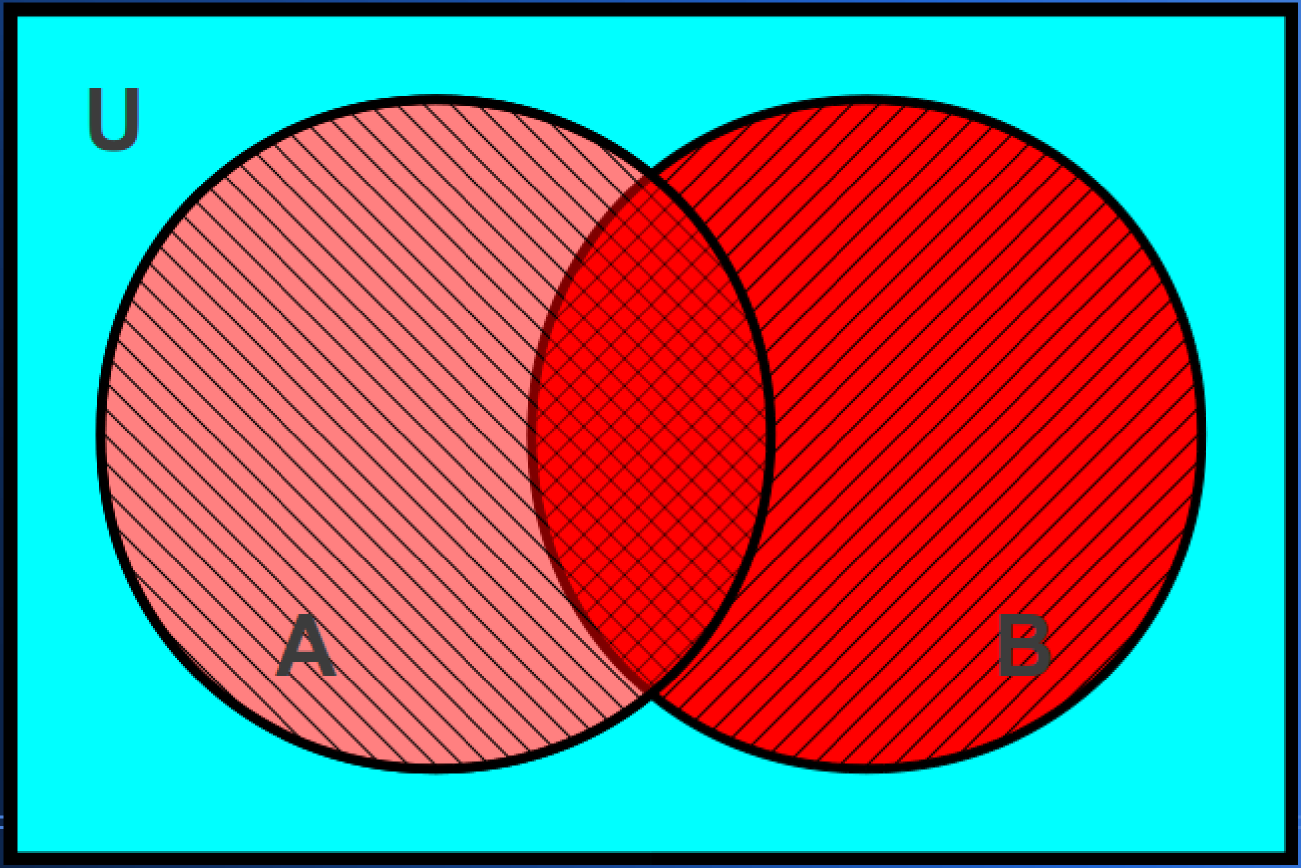
(checkered)



Mathematical Background

U the universal class

Algebra of sets



A ∪ B

(reddish)

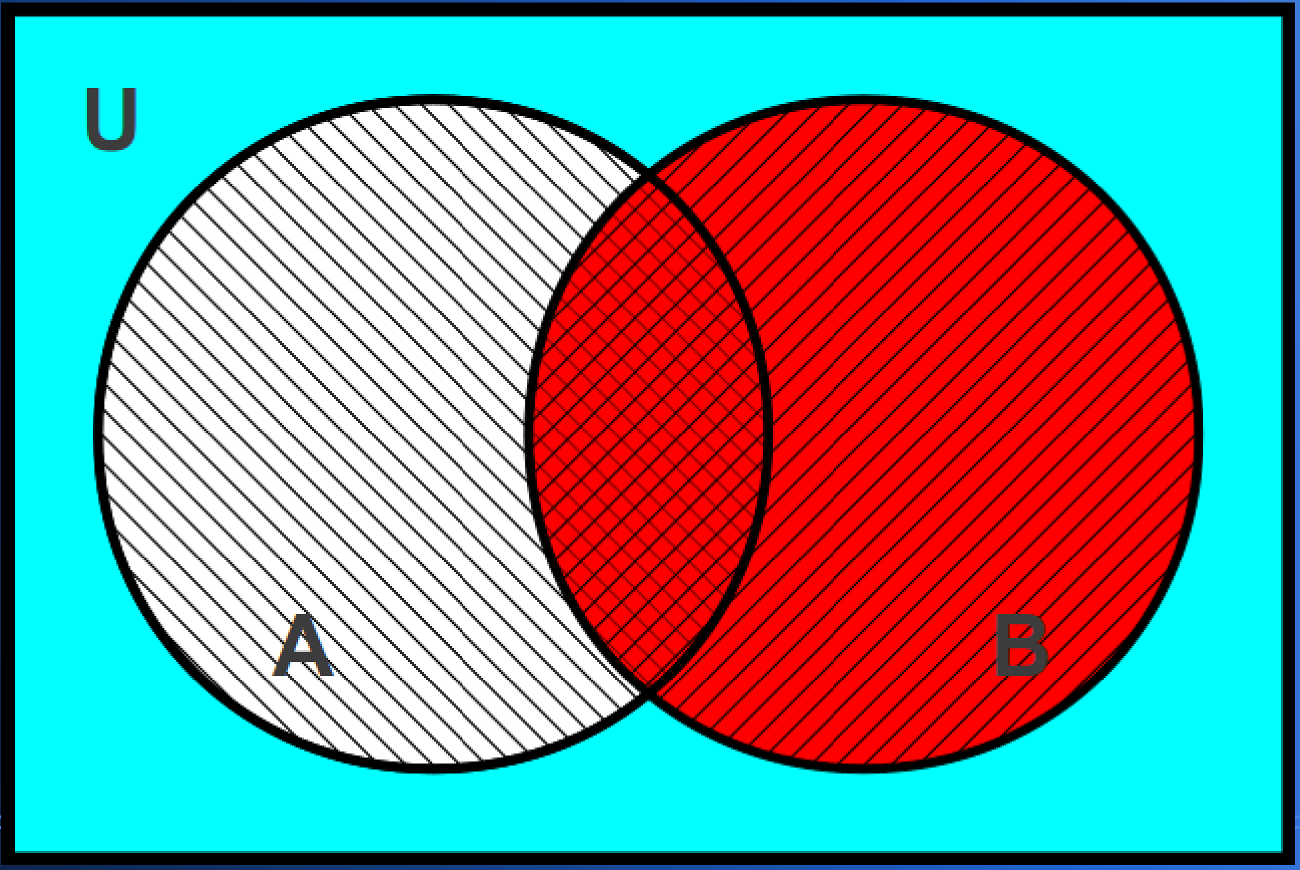
Mathematical Background

U the universal class

Algebra of sets

A – B

(white)



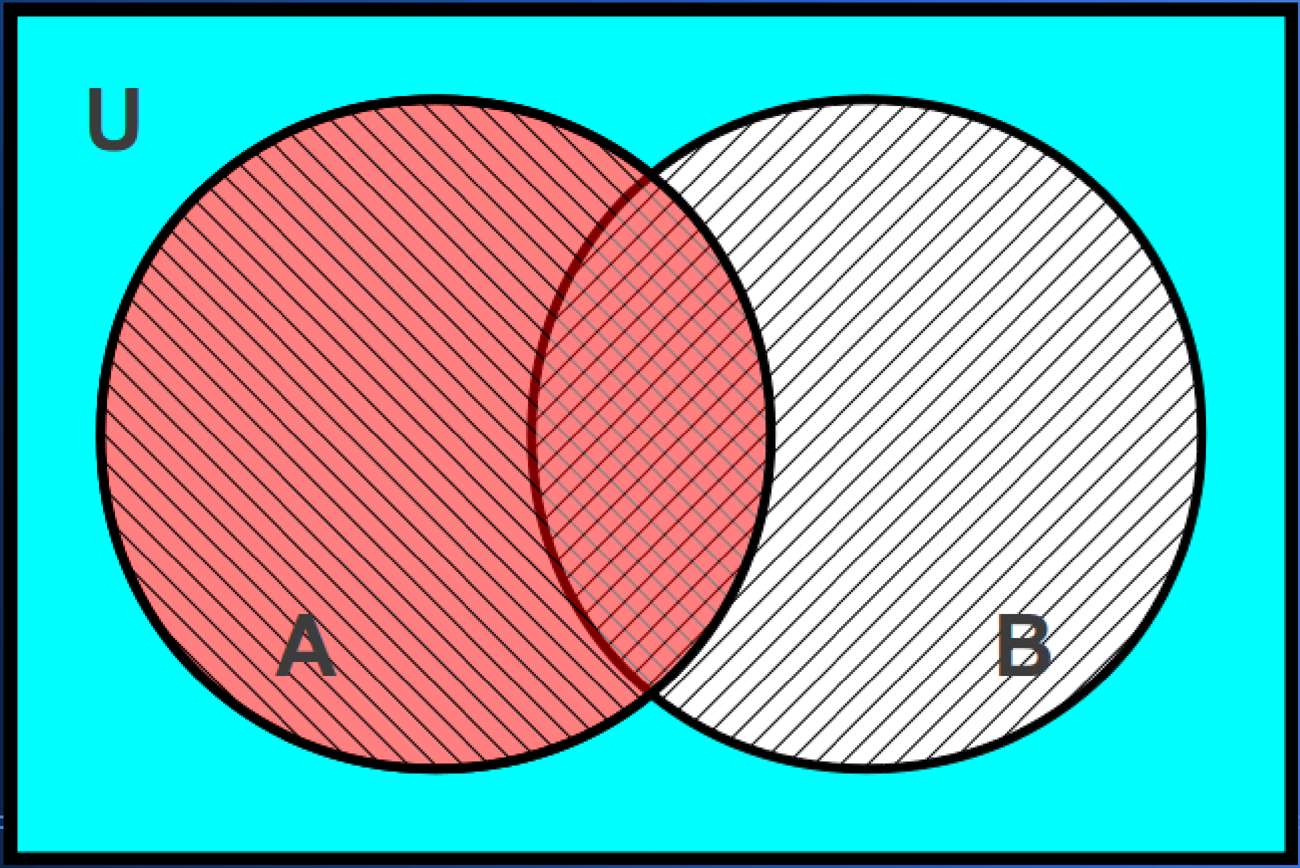
Mathematical Background

U the universal class

Algebra of sets

B – A

(white)



Mathematical Background

U the universal class

the union of sets

Mathematical Background

U the universal class

the union of sets

$(\forall A \in U)(\forall B \in U)(A \cup B = \{x \in U \mid x \in A \vee x \in B\})$

Mathematical Background

U the universal class

the union of sets

$(\forall A \in U)(\forall B \in U)(A \cup B = \{x \in U \mid x \in A \vee x \in B\})$

$A \cup B = \{x \mid x \in A \vee x \in B\}$

Mathematical Background

U the universal class

the union of sets

$(\forall A \in U)(\forall B \in U)(A \cup B = \{x \in U \mid x \in A \vee x \in B\})$

$A \cup B = \{x \mid x \in A \vee x \in B\}$

$(\forall x \in U)(x \in A \cup B \equiv x \in A \vee x \in B)$

Mathematical Background

U the universal class

the intersection of sets

Mathematical Background

U the universal class

the intersection of sets

$(\forall A \in U)(\forall B \in U)(A \cap B = \{x \in U \mid x \in A \wedge x \in B\})$

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

$(\forall x \in U)(x \in A \cap B \equiv x \in A \wedge x \in B)$

Mathematical Background

U the universal class

the difference of sets

$(\forall A \in U)(\forall B \in U)(A - B = \{x \in U \mid x \in A \wedge x \notin B\})$

$A - B = \{x \mid x \in A \wedge x \notin B\}$

$(\forall x \in U)(x \in A - B \equiv x \in A \wedge x \notin B)$

Mathematical Background

U the universal class

the complement of a set

$(\forall B \in U)(-B = \{x \in U \mid x \notin B\})$

$-B = \{x \mid x \notin B\}$

$(\forall x \in U)(x \in -B \equiv x \notin B)$

Mathematical Background

Cardinality of a set

Mathematical Background

Cardinality of a set

#(A)

Mathematical Background

Cardinality of a set

#(A)   the number of elements of A

Mathematical Background

Cardinality of a set

#(A)   the number of elements of A

For a finite set A, one can count elements of A.

Mathematical Background

Cardinality of a set

#(A) the number of elements of A

For a finite set A, one can count elements of A.

For infinite sets, it's tricky.

Mathematical Background

Cardinality of a set

For infinite sets, it's tricky.

Mathematical Background

Cardinality of a set

For infinite sets, it's tricky.

Sets A and B have the same cardinality

Mathematical Background

Cardinality of a set

For infinite sets, it's tricky.

Sets A and B have the same cardinality iff

Mathematical Background

Cardinality of a set

For infinite sets, it's tricky.

Sets A and B have the same cardinality iff
there is a 1-1 function with domain A and range B.

Mathematical Background

Cardinality of a set

Sets A and B have the same cardinality iff there is a 1-1 function with domain A and range B.

Mathematical Background

Cardinality of a set

Sets A and B have the same cardinality iff there is a 1-1 function with domain A and range B.

Cardinality of 0 is

Mathematical Background

Cardinality of a set

Sets A and B have the same cardinality iff there is a 1-1 function with domain A and range B.

Cardinality of 0 is 0

Mathematical Background

Cardinality of a set

Sets A and B have the same cardinality iff there is a 1-1 function with domain A and range B.

Cardinality of 0 is 0

Cardinality of N is

Mathematical Background

Cardinality of a set

Sets A and B have the same cardinality iff there is a 1-1 function with domain A and range B.

Cardinality of 0 is 0

Cardinality of N is N

Mathematical Background

Cardinality of a set

Cardinality of N is N

What is the cardinality of Even?

Mathematical Background

Cardinality of a set

Cardinality of N is N

What is the cardinality of Even?

N = {0, 1, 2, 3, …}

Mathematical Background

What is the cardinality of Even?

N = {0, 1, 2, 3, …}

Mathematical Background

What is the cardinality of Even?

N = {0, 1, 2, 3, …}

Even = {0, 2, 4, 6, ...}

Mathematical Background

What is the cardinality of Even?

$N = \{0, 1, 2, 3, \ldots\}$

$Even = \{0, 2, 4, 6, \ldots\} = \{2\times0, 2\times1, 2\times2, 2\times3, \ldots\}$

Mathematical Background

What is the cardinality of Even?

$N = \{0, 1, 2, 3, \ldots\}$

$Even = \{0, 2, 4, 6, \ldots\} = \{2\times0, 2\times1, 2\times2, 2\times3, \ldots\}$

So, $\#(Even) = \#(N)$

Mathematical Background

What is the cardinality of Even?

N = {0, 1, 2, 3, …}

Even = {0, 2, 4, 6, …} = {2×0, 2×1, 2×2, 2×3, …}

So, #(Even) = #(N) = N

Mathematical Background

So, #(Even) = #(N) = N

Mathematical Background

So, #(Even) = #(N) = N

although Even is a **proper** subset of N

Mathematical Background

So, #(Even) = #(N) = N

although Even is a **proper** subset of N
so Even should be "smaller" than N.

Mathematical Background

So, #(Even) = #(N) = N

although Even is a **proper** subset of N
so Even should be “smaller” than N.

This “paradox” yields the definition of infinite set.

Mathematical Background

This “paradox” yields the definition of infinite set.

Mathematical Background

This “paradox” yields the definition of infinite set.

A set A is infinite

Mathematical Background

This “paradox” yields the definition of infinite set.

A set A is infinite iff

Mathematical Background

This “paradox” yields the definition of infinite set.

A set A is infinite iff

$(\exists B)(B \subseteq A \wedge B \neq A \wedge \#(B) = \#(A))$

Mathematical Background

This “paradox” yields the definition of infinite set.

A set A is infinite iff

$(\exists\ B)(B \subseteq A \wedge B \neq A \wedge \#(B) = \#(A))$

read: A has the same number of elements as some of its proper subsets

Mathematical Background

This "paradox" yields the definition of infinite set.

A set A is infinite iff

$(\exists B)(B \subseteq A \wedge B \neq A \wedge \#(B) = \#(A))$

read: A has the same number of elements as some of its proper ("smaller" that is) subsets

Mathematical Background

Ordered pair

Mathematical Background

Ordered pair (a, b)

Mathematical Background

Ordered pair (a, b) {{a, b}, a}

Mathematical Background

Ordered pair (a, b) {{a, b}, a}

Cross product $A \times B$

Mathematical Background

Ordered pair (a, b) {{a, b}, a}

Cross product $A \times B$

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

Mathematical Background

Ordered pair (a, b) $\{\{a, b\}, a\}$

Cross product $A \times B$

$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$$

$$\{2,3\} \times \{a,b,c\} = \{(2, a),(2,b),(2,c),(3,a),(3,b),(3,c)\}$$

Mathematical Background

$$A \times B = \{(a, b) | a \in A \wedge b \in B\}$$

$$\{2,3\} \times \{a,b,c\} = \{(2, a),(2,b),(2,c),(3,a),(3,b),(3,c)\}$$

Mathematical Background

$A \times B = \{(a, b) | a \in A \wedge b \in B\}$

$\{2,3\} \times \{a,b,c\} = \{(2, a),(2,b),(2,c),(3,a),(3,b),(3,c)\}$

Binary relation R on a sets A and B

Mathematical Background

$A \times B = \{(a, b) | a \in A \wedge b \in B\}$

$\{2,3\} \times \{a,b,c\} = \{(2, a),(2,b),(2,c),(3,a),(3,b),(3,c)\}$

Binary relation R on sets A and B

$R \subseteq A \times B$

Mathematical Background

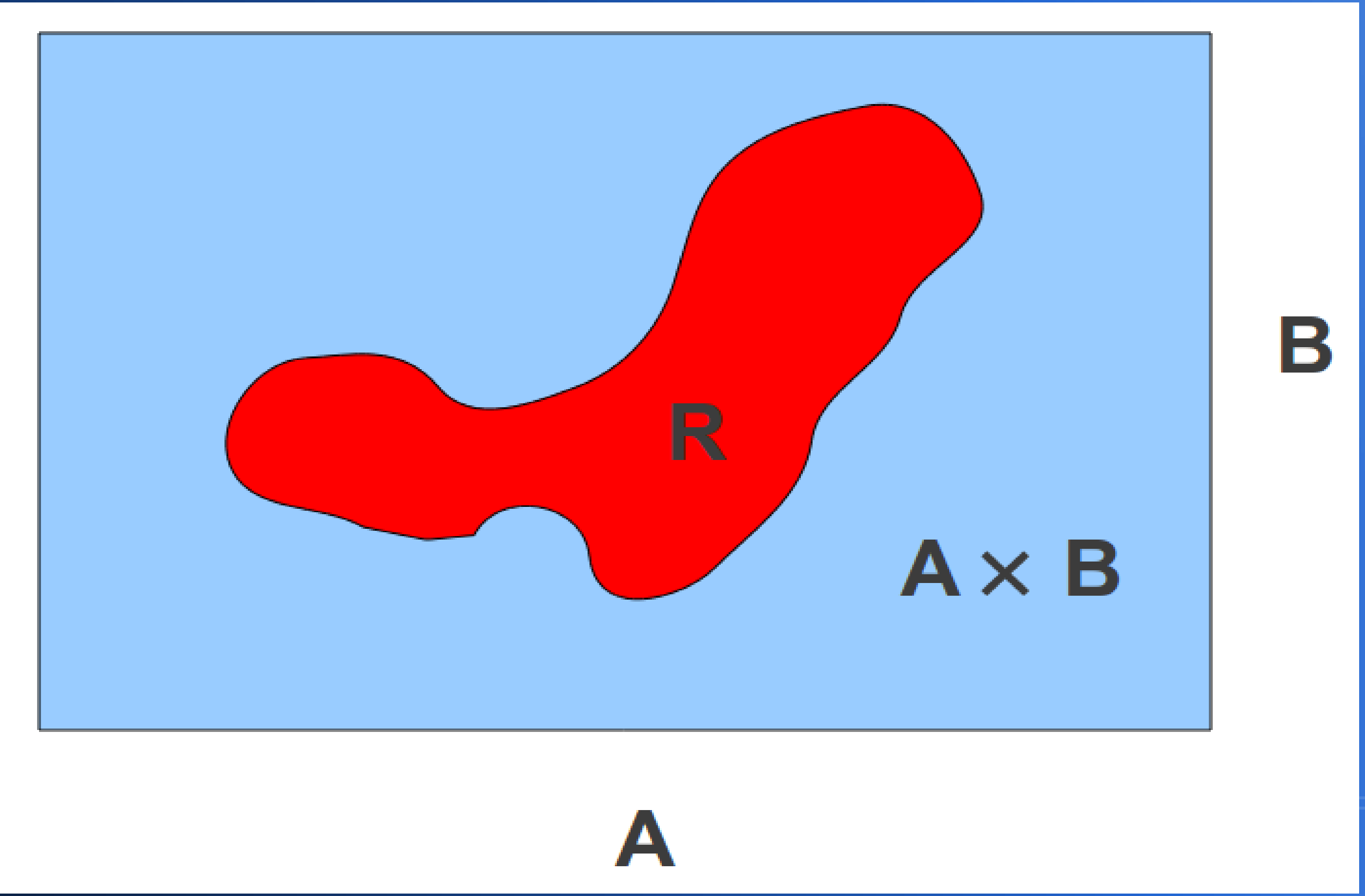
Binary relation R on sets A and B

$$R \subseteq A \times B$$

Mathematical Background

Binary relation R on sets A and B

$R \subseteq A \times B$

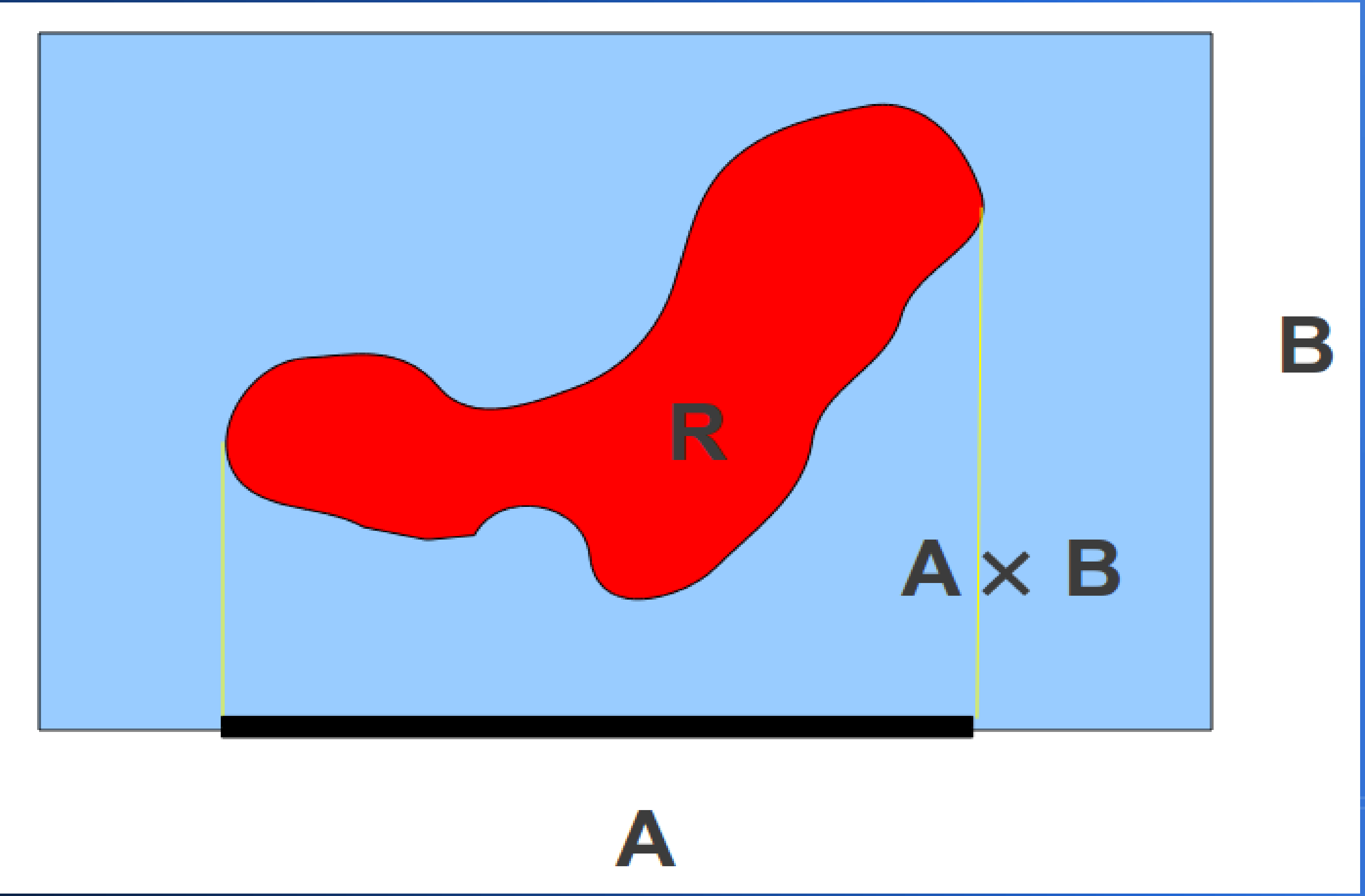


Mathematical Background

Binary relation R on sets A and B

$R \subseteq A \times B$

Dm(R)

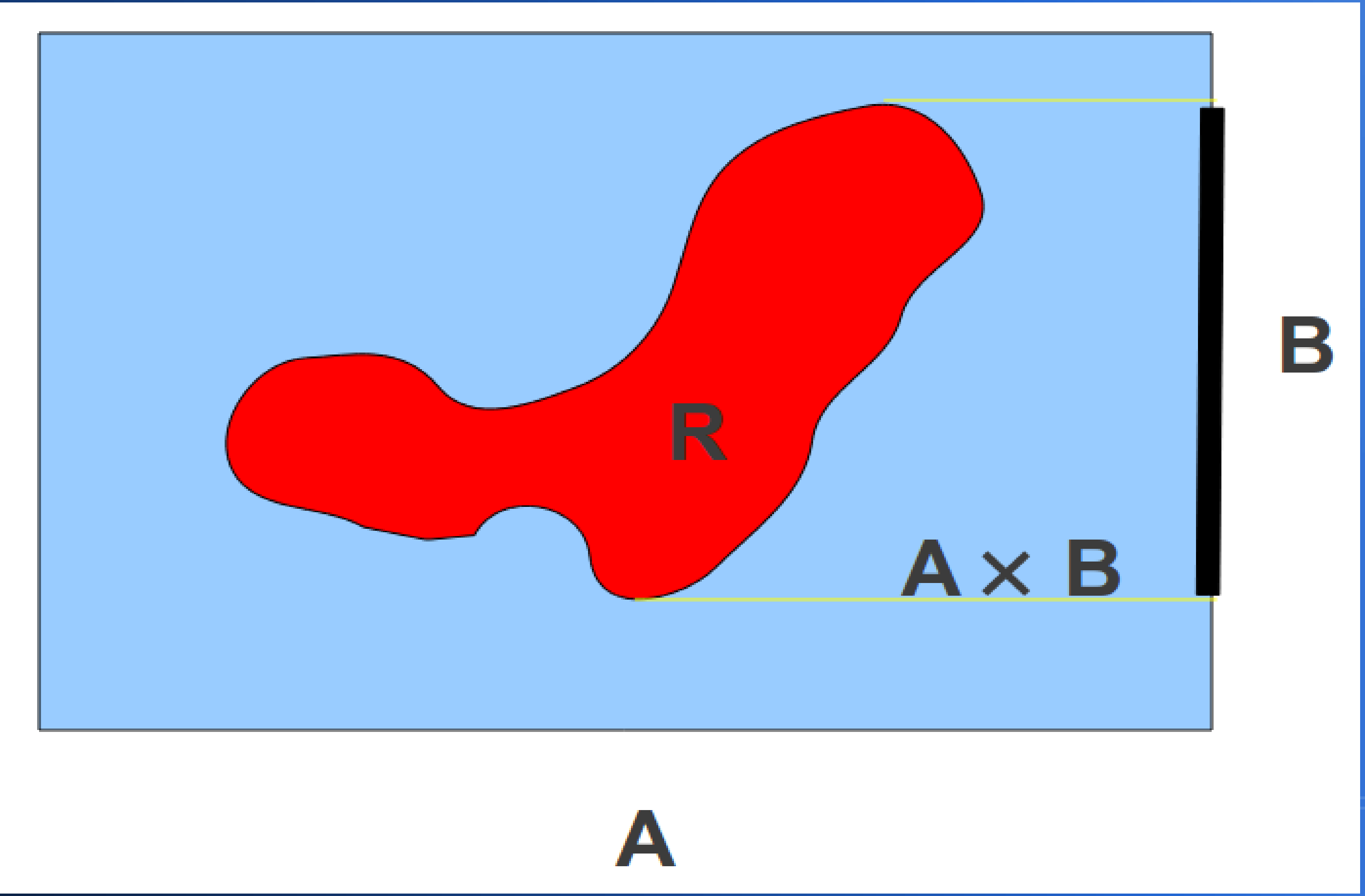


Mathematical Background

Binary relation R on sets A and B

$R \subseteq A \times B$

Rg(R)



Mathematical Background

Binary relation R on sets A and B

$R \subseteq A \times B$

Binary relation R on a set S

Mathematical Background

Binary relation R on sets A and B

$R \subseteq A \times B$

Binary relation R on a set S

$R \subseteq S \times S$

Mathematical Background

Tuple

Mathematical Background

Tuple (a, b, …, w)

Mathematical Background

Tuple (a, b, …, w)

An ordered pair is a 2-tuple

Mathematical Background

Tuple (a, b, …, w)

An ordered pair is a 2-tuple

An ordered three is a 3-tuple

Mathematical Background

Tuple (a, b, …, w)

An ordered pair is a 2-tuple

An ordered three is a 3-tuple

An ordered four is a 4-tuple

Mathematical Background

Tuple (a, b, …, w)

Mathematical Background

Tuple (a, b, …, w)

Then they are n-tuples.

Mathematical Background

Tuple (a, b, …, w)

Then they are n-tuples.

n-ary relation R

Mathematical Background

Tuple (a, b, …, w)

Then they are n-tuples.

n-ary relation R is a set of n-tuples

Mathematical Background

Tuple (a, b, …, w)

Then they are n-tuples.

n-ary relation R is a set of n-tuples

$$R \subseteq A_1 \times A_2 \times \ldots \times A_n$$

Mathematical Background

And so it goes …

Mathematical Background

And so it goes …

Binary relations can be reflexive, symmetric, transitive, antitransitive.

Mathematical Background

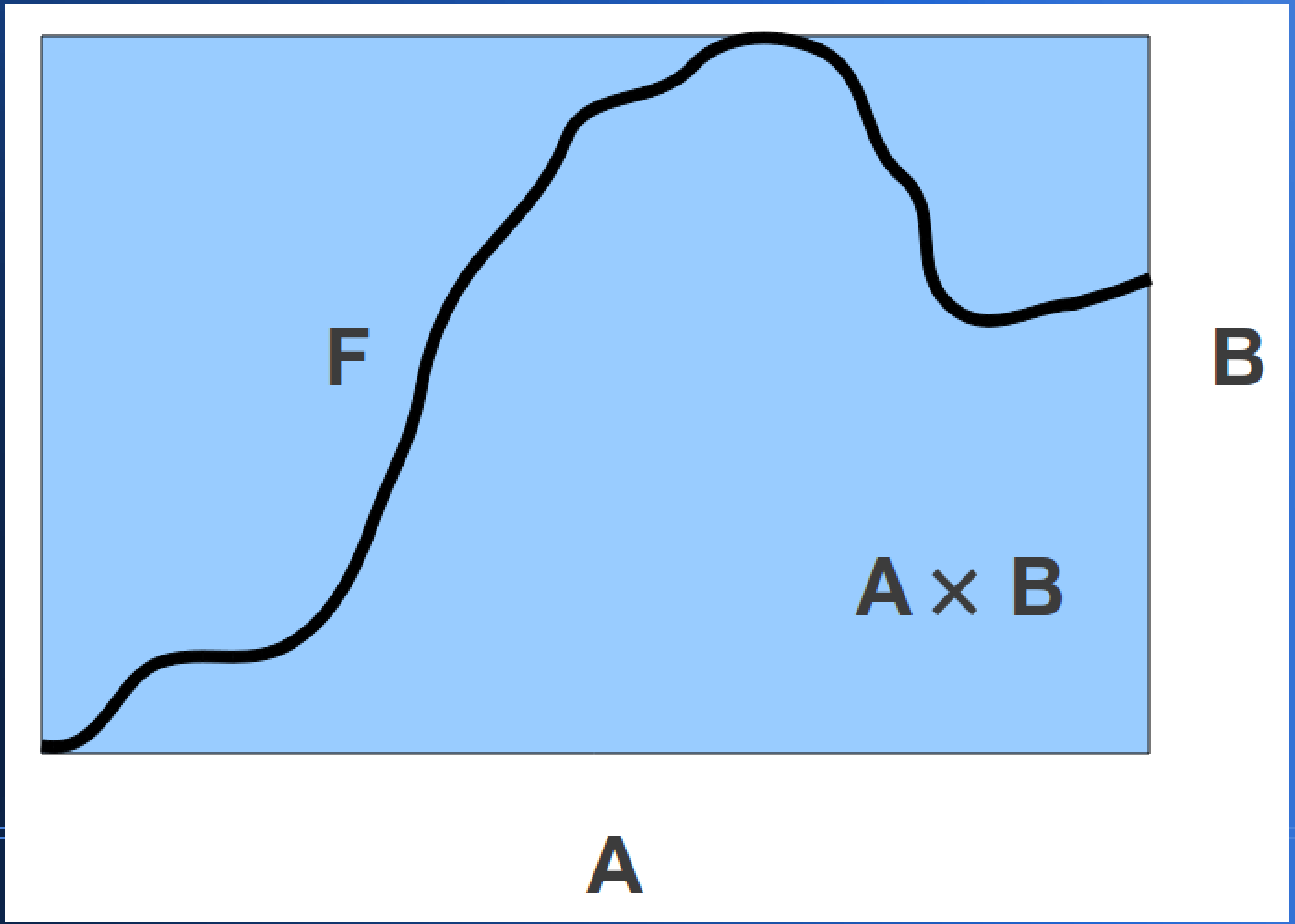
And so it goes …

Binary relations can be reflexive, symmetric, transitive, antitransitive.

They can also be functions.

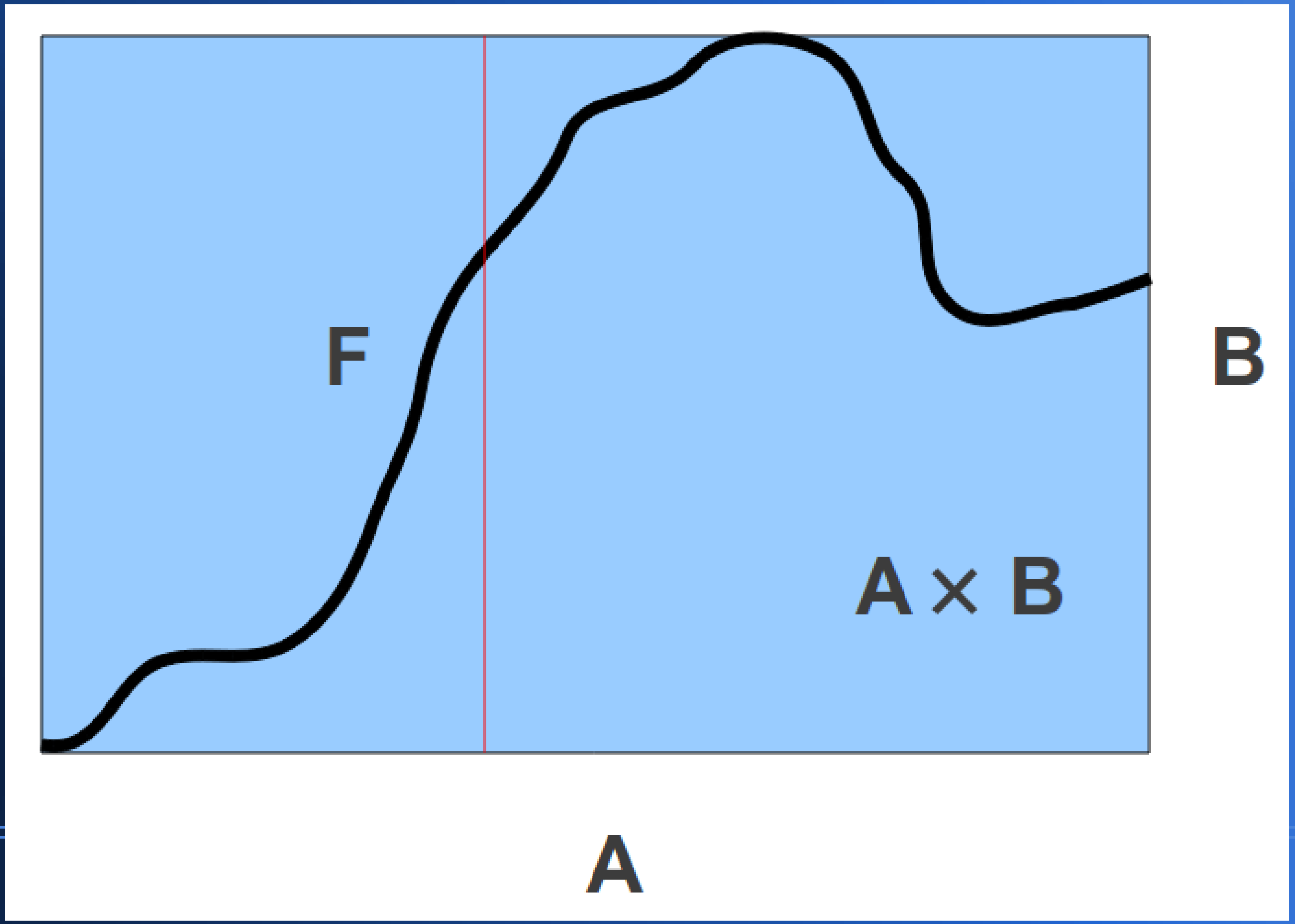
Mathematical Background

They can also be functions.



Mathematical Background

They can also be functions.



Mathematical Background

They can also be functions.

Domain of F is Dm(F),

Mathematical Background

They can also be functions.

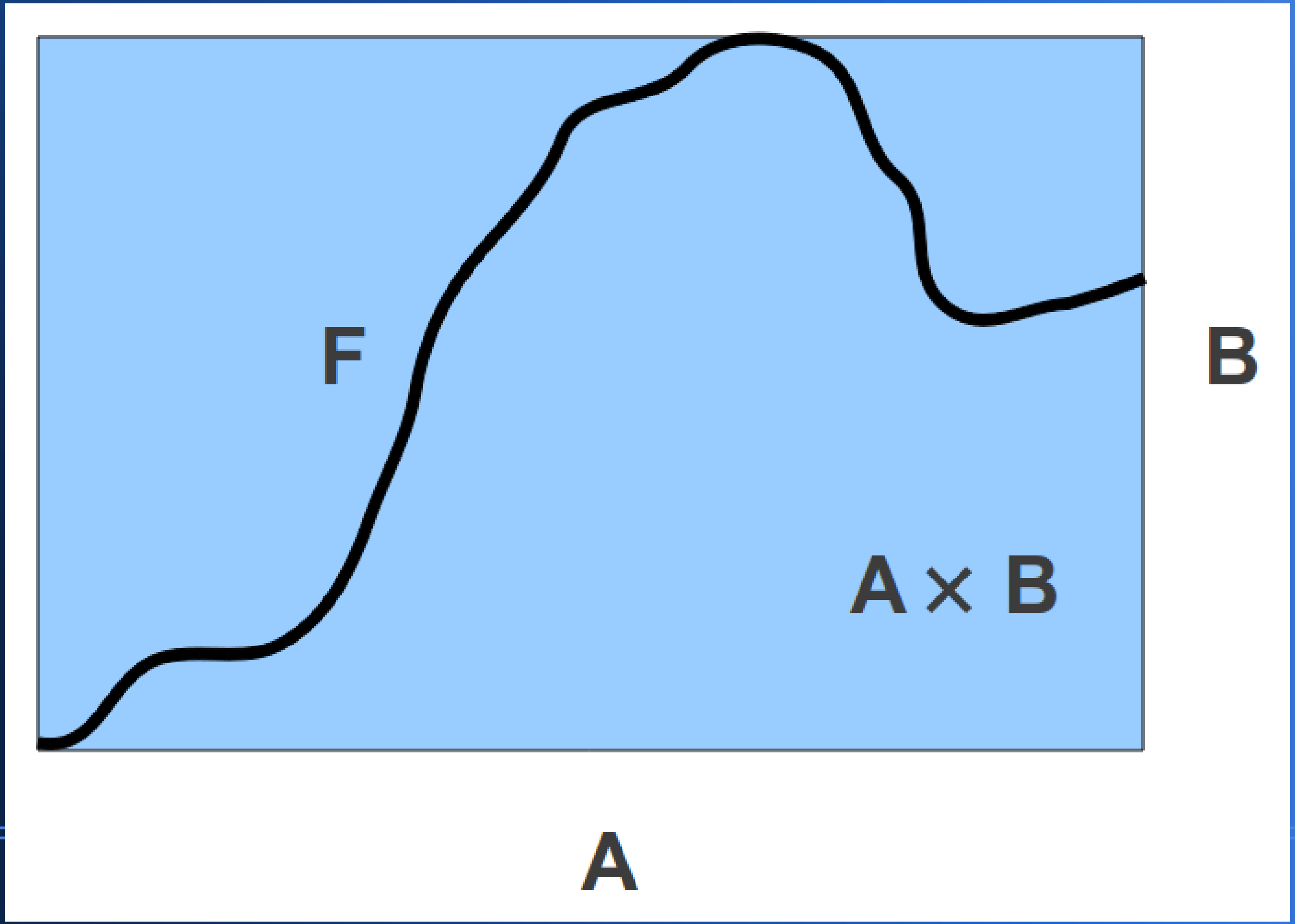
Domain of F is Dm(F)

Range of F is Rg(F)

Mathematical Background

They can also be functions.

Dm(F) = A

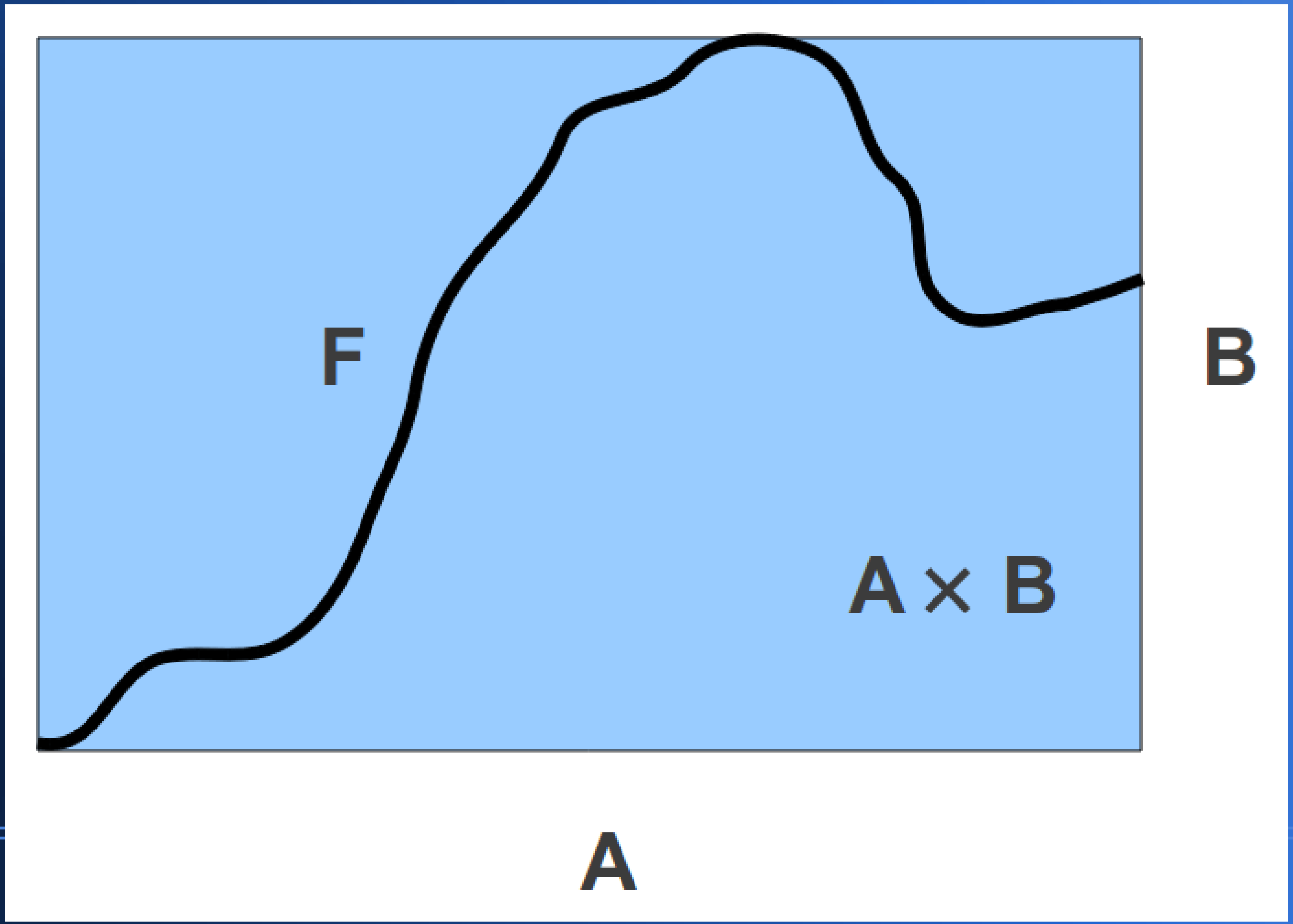


Mathematical Background

They can also be functions.

Dm(F) = A

Rg(F) = B



Mathematical Background

1 – 1 functions have (unique) inverses.

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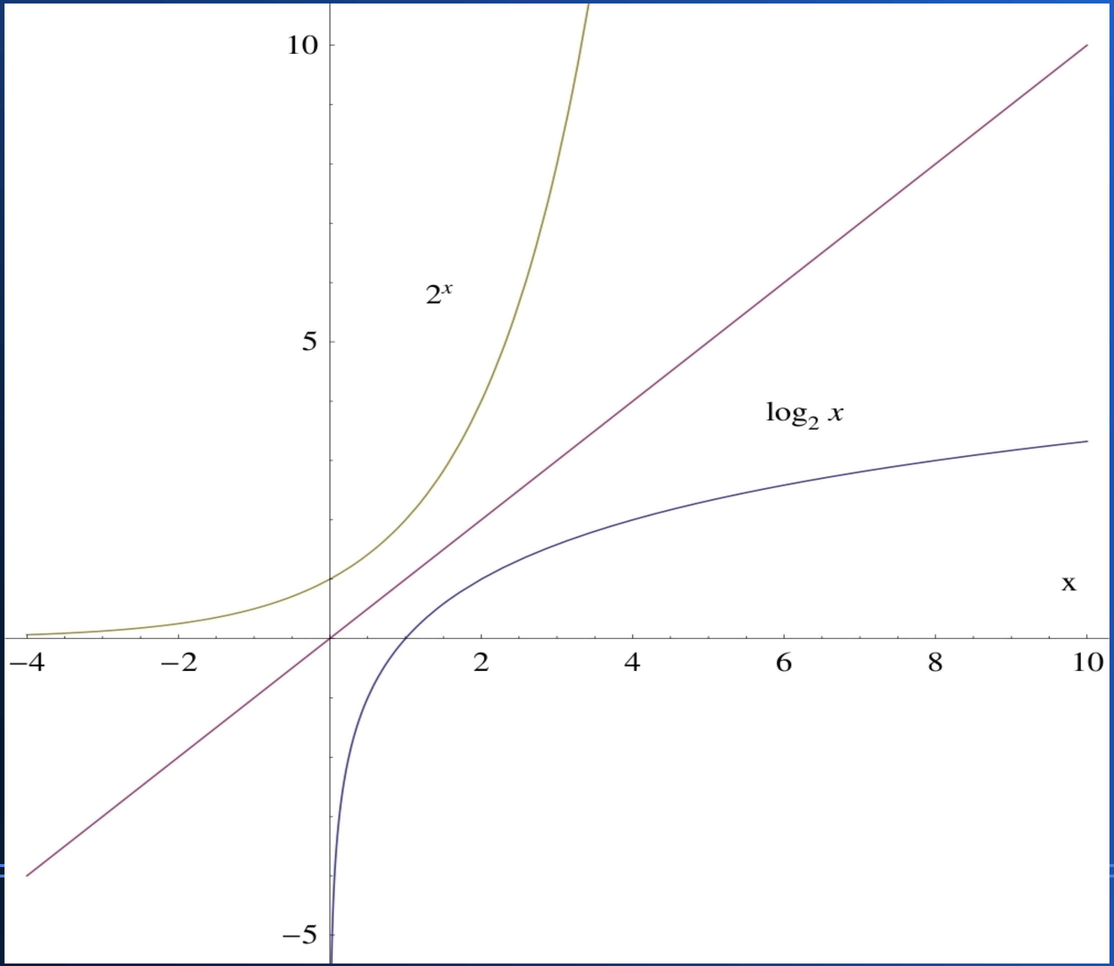

Mathematical Background

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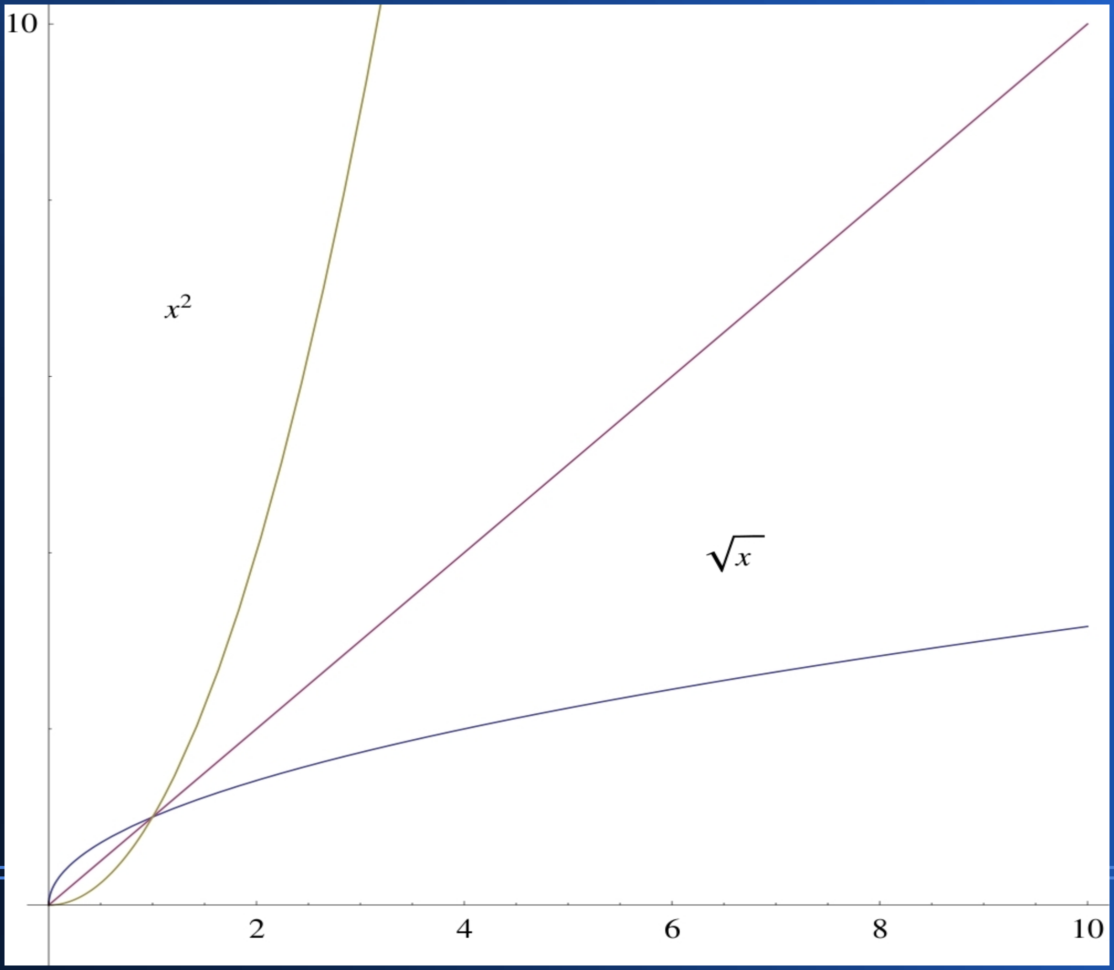

Mathematical Background

Make sure to review MAT 281