A Comment on Textbook Analysis of MergeSort

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There is a derivation of the worst-case number of comparisons of `MergeSort` in a textbook [BG00], page 177:

$$\begin{aligned} W(n) &= \sum_{d=0}^{D-2} \left(n - 2^d\right) + (n - B)/2 \\ &= n(D-1) - 2^{D-1} + 1 + (n - B)/2 \\ &= n\,D - 2^D + 1. \end{aligned} \tag{4.6}$$

Because D is rounded up to an integer and occurs in the exponent, it is hard to tell how Equation (4.6) behaves between powers of 2.

for $D = \lceil \lg(n+1) \rceil$ and $n = 2^k$ for some k, which ends with a remark that "it is hard to tell how Equation (4.6) behaves between powers of 2."

This suggests that the authors were not aware that the formula

$$W(n) = n\lceil \lg(n+1) \rceil - 2^{\lceil \lg(n+1) \rceil} + 1 \tag{1}$$

holds for any $n \geq 1$, and not just for $n = 2^k$.

The formula (1) can be simplified to:

$$W(n) = n\lceil \lg n \rceil - 2^{\lceil \lg n \rceil} + 1 \tag{2}$$

or to:

$$W(n) = \sum_{i=1}^{n} \lceil \lg i \rceil. \tag{3}$$

The formula (1) was proven in file `Worst-caseMergesort.pdf` and in class for every $n \geq 1$. The formulas (2) and (3) are proven in file `Knuth-Suchenek_formulas_sums_of_floors_ceilings_logs.pdf`.

References

[BG00] Sara Baase and Allen Van Gelder. *Computer Algorithms; Introduction to Design & Analysis.* Asddison Wesley, 3rd edition, 2000.