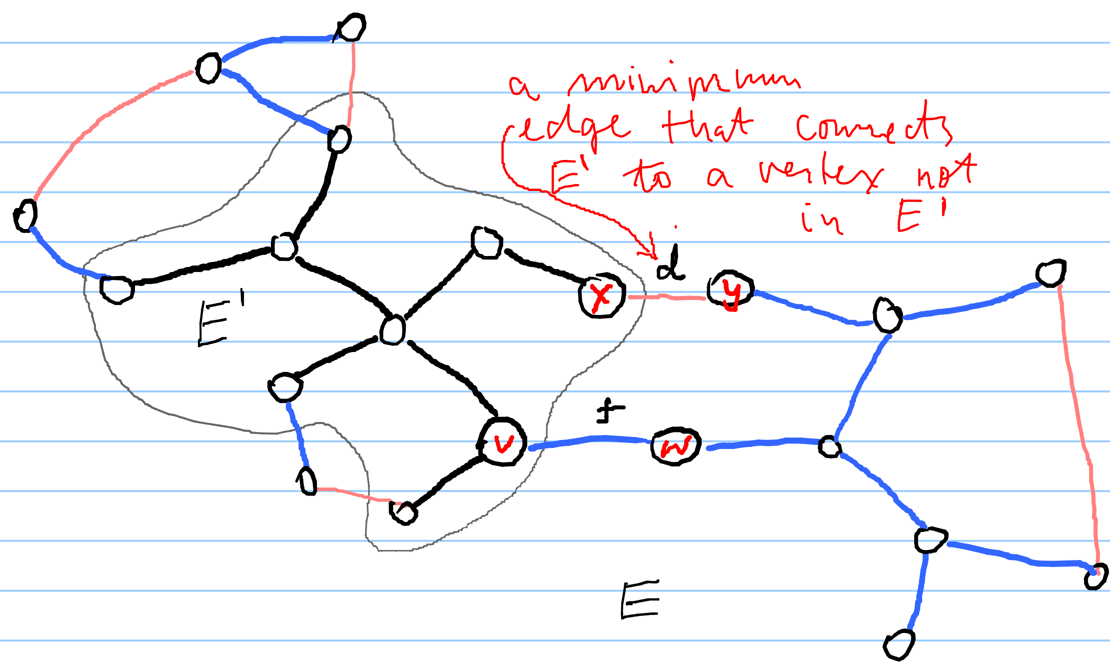
Minimum spanning tree (Dijkstra – Prim)

Note Title 4/17/2012



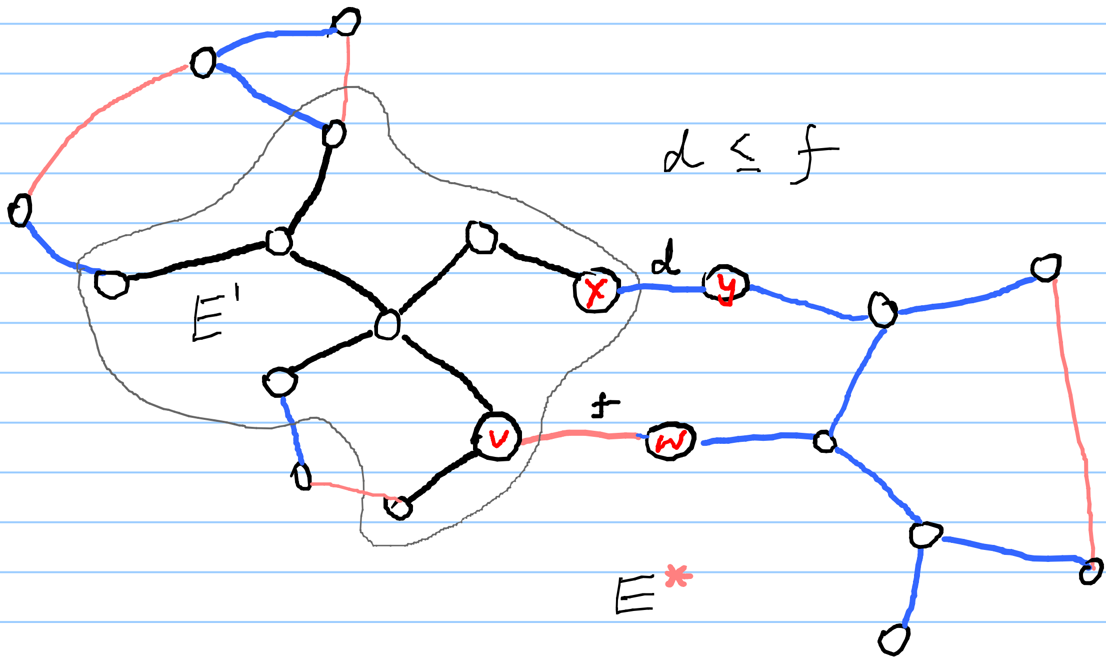
E – a MST

E' – a tree that is a subtree of E

edges of E: black, blue

edges of E': black

other edges (not in E or E'): pink



E^* - a MST

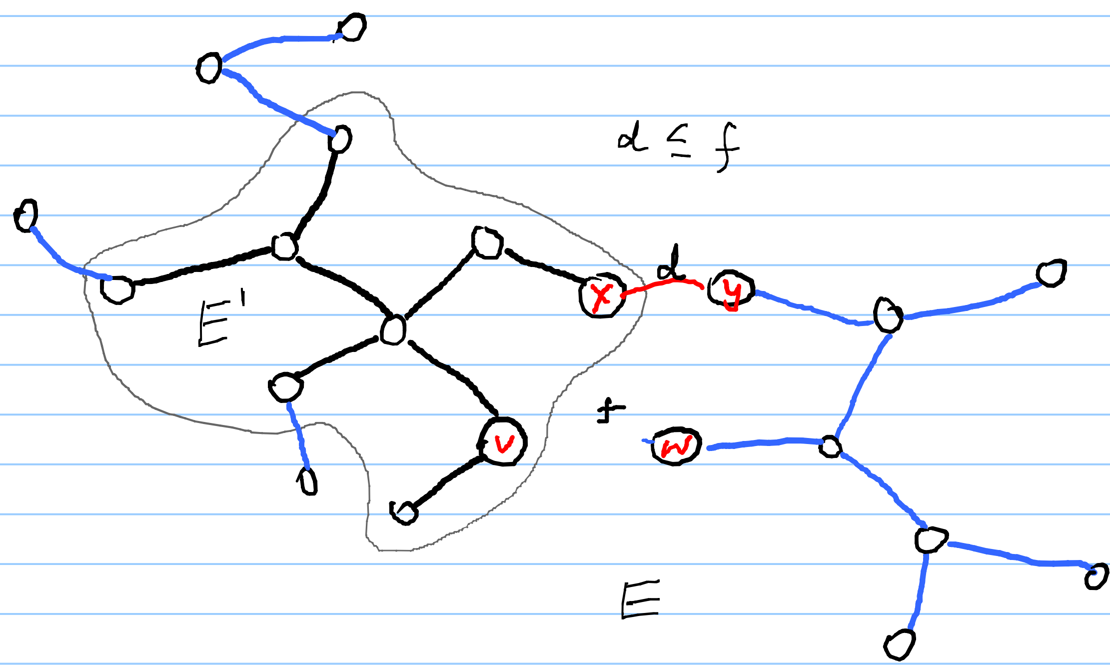
E' - a tree that is a subtree of E^*

edges of E^*: — —

edges of E': —

other edges (not in E^* or E'): —

So, $d = f$ and E^* is a MST that contains E' and edge (x, y).

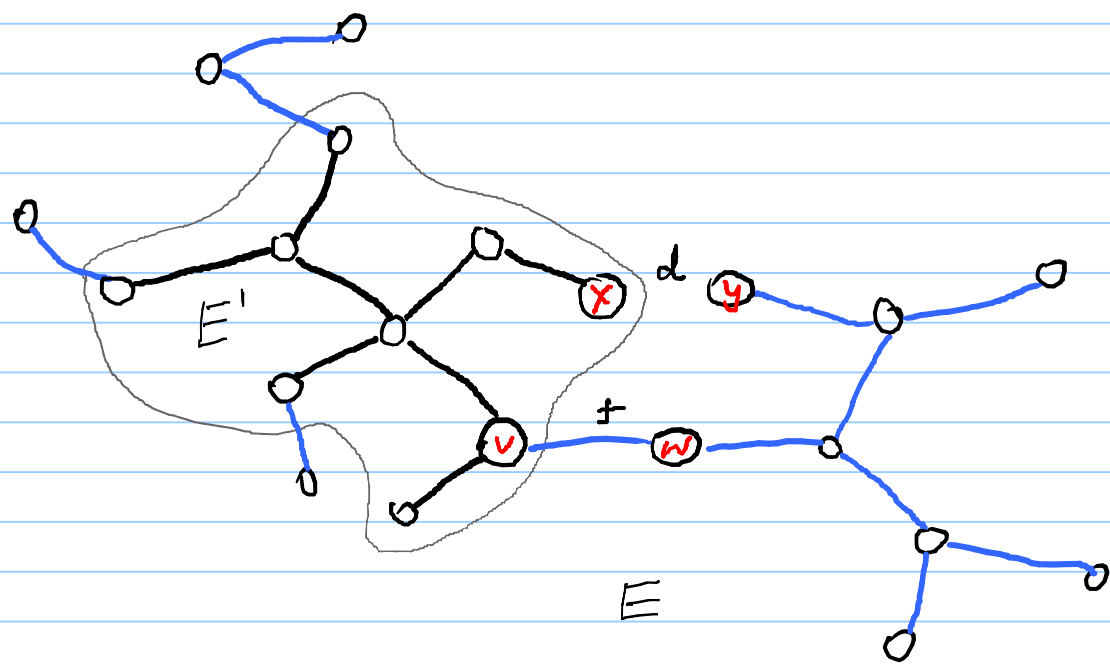


E – a MST

E' – a tree that is a subtree of E

edges of E: black, blue

edges of E': black



E – a MST

E' – a tree that is a subtree of E

edges of E: — —

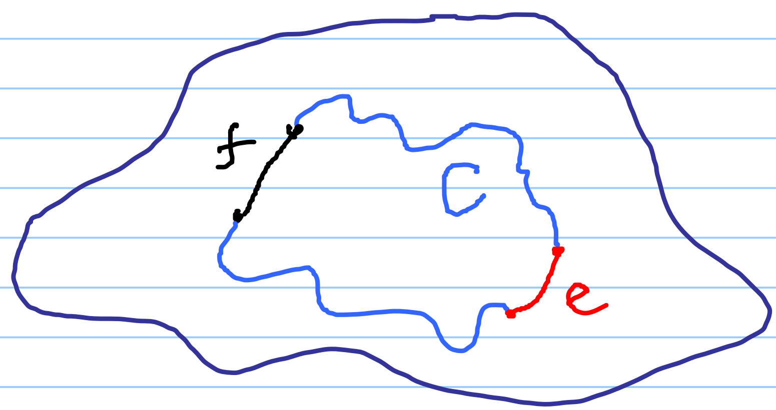
edges of E': —

Note. Here is a useful construction that converts one tree T onto another tree T'.

1. Add an edge $e \notin T$ to T. This results in a set $T \cup \{e\}$ that has a cycle. Let C be the set of edges of that cycle.
2. Find an edge $f \in C$, with $f \neq e$. Alternatively, one can say, $f \in C \setminus \{e\}$.

Of course, $f \in T$ because $C \subseteq T \cup \{e\}$ and $C \setminus \{e\} \subseteq T$.

3. Remove f from $T \cup \{e\}$. This results in $T' = (T \cup \{e\}) \setminus \{f\}$.



Since T is connected, $T \cup \{e\}$ is connected, too.

Since f is an edge in a cycle in $T \cup \{e\}$, $T' = (T \cup \{e\}) \setminus \{f\}$ is connected.

Since T is a tree, and T' is a connected graph with the same number of vertices (n) as T and the same number of edges $(n-1)$ $\Rightarrow$ T, T' is a tree.