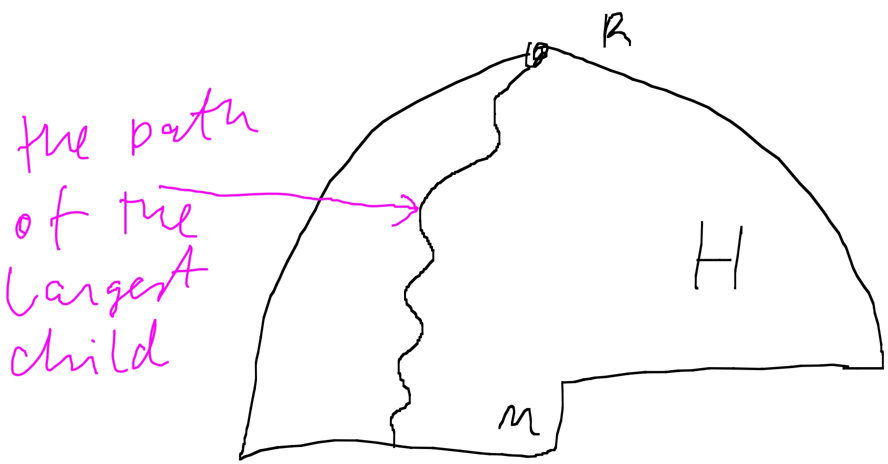
Note re. proof of $\lfloor \lg n \rfloor + \lfloor \lg \lg n \rfloor + 1$ for AHS

Note Title 4/25/2014

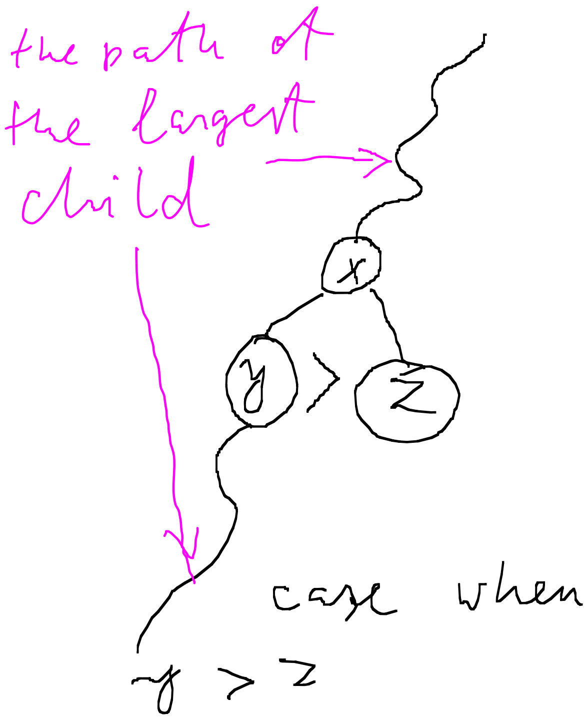
For the proof of the fact that accelerated FixHeap performs no more than $\lfloor \lg n \rfloor + \lfloor \lg \lg n \rfloor + 1$ comparisons see file Proof_comps_AccHeapSort.nb

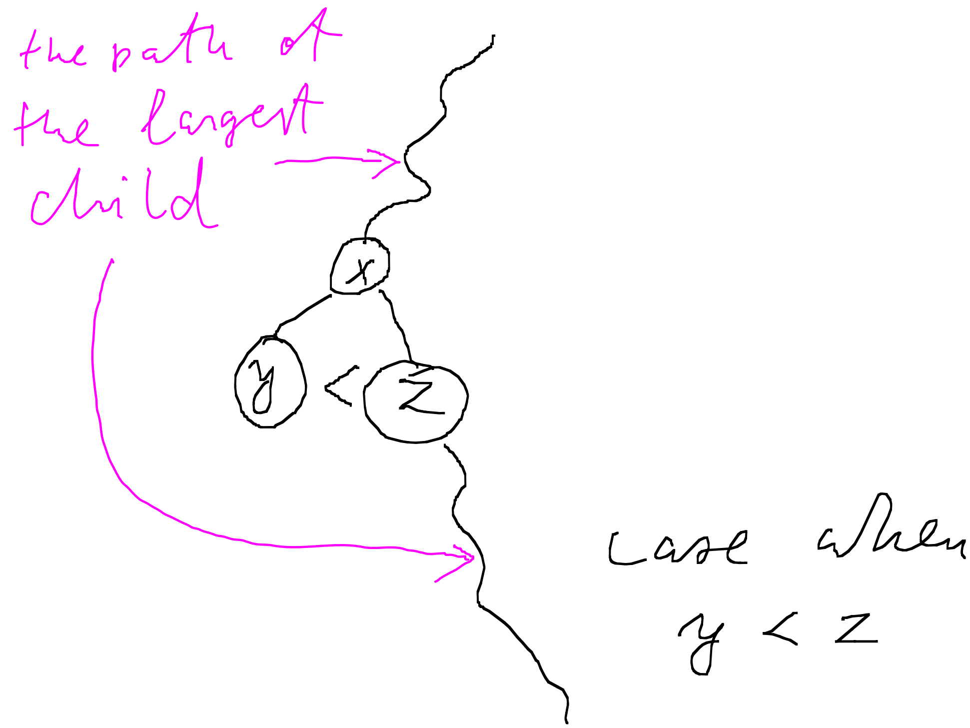
or Proof_comps_AccHeapSort.pdf.

Let H be an almost heap on n nodes. "Almost" means here that H satisfy all the properties of heap except, perhaps, that its root R is less than the largest of its children (if it has any). Accelerated fixheap will demote R along the path of the largest child

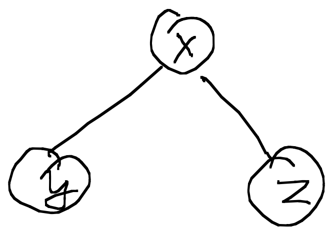


To find the path of the largest child in H, the number of comparisons needed is equal to the length of that path, because it requires comparison of children of each node in that path, as the following picture shows.





The maximum numbers of trios



along any path from the root R of H down is equal to the length L of that path if the last parent along that path has 2 children or is L-1 if the last parent has one child only (simply because in the latter case there is nothing ~~to~~ to compare that only child with).

In any case, the number of comps is not greater than L.

Since the length of any path in H from the root R down is not greater than the depth D of H, and since $D = \lfloor \lg_2 q \rfloor$, the number of said comparisons is not greater than $\lfloor \lg n \rfloor$.

This note is not a complete proof of the $\lfloor \lg n \rfloor + \lfloor \lg \lg n \rfloor + 1$ formula. For the complete proof consult the main file referenced on the first page of this one.