Minimal Models for Closed World Data Bases

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Abstract

This paper investigates a consistent version cwa_S Reiter's closed world assumption cwa. It proves (cf. theorem 4.11) that for purely relational languages cwa_S is $\forall$-complete with respect to minimal semantics, i.e. for every $\forall$-sentence φ and for every $\forall$-theory Σ,

$$\varphi \in cwa_S(\Sigma) \text{ iff } \Sigma \vdash_{min} \varphi.$$

Moreover, it relates cwa_S to other known syntactic characterization of minimal semantics: Minker's *GCWA*.

1 Introduction

Reiter's closed world assumption cwa has been introduced in [Rei78]. It may be defined as follows.

> If an atomic sentence φ is not implied by the information contained in a data base then the negation of φ is asserted.

In some cases cwa has led to a contradiction when applied to indefinite data bases, where disjunctions of atomic sentences are allowed. E.g. $cwa(\varphi \vee \psi)$ entails both $\neg\varphi$ and $\neg\psi$, that is to say, $\neg(\varphi \vee \psi)$. Minker has proposed in [Min82] a weaker version *GCWA* of cwa. It asserts the negation of an atomic sentence φ only if for every positive (i.e. without appearance of negation) $\forall$-sentence ψ, which is non-derivable from a data base, $\varphi \vee \psi$ cannot be derived from that data base. It has been proved (cf. [She88], thm. 32.5) that *GCWA* coincides with cwa in all data bases for which cwa is consistent.

Another version of closed world assumption, cwa_S, has been proposed in [Suc87]:

> A sentence φ is asserted if, and only if, it does not enlarge the set of positive consequences of the data base.

Minimal model semantics, based on Lyndon's relation of enlargement of [Lyn59], seems to reflect the intended meaning of closed world assumption. Therefore, a syntactic system completely characterizing this semantics may be recognized as a proper formulation of cwa. We will demonstrate that cwa_S provides such characterization for the class of all $\forall$-sentences (or, equivalently, for the set of all clauses). Other known solutions to the problem of characterization do not have this completeness property.

2 Negative forcing - prerequisites

In this section we quote some definitions and results from [Suc88]. Most of them belong to the standard first-order Model Theory, and can be found (in equivalent form) in classical texts, e.g. [Bar78] (particularly chapters: [Kei78] and [Mac78]), and [Kei73].

We consider a first-order countable language L, with an infinite set C of constants, without equality symbol $=$, and without function symbols. We follow the notation of [Bar78], Chap. A.1, A.2, A.4, and B.1. We denote first-order structures for L by $\mathcal{A}, \mathcal{B}, \mathcal{M}, \mathcal{N}, ...$, and their universes by $A, B, M, N, ...$, respectively. A formula of L is *positive* iff it is built of atomic formulae using exclusively $\wedge, \vee, \forall, \exists$. A formula is *negative* iff it is a negation of a positive formula. An $\forall$-formula is a formula of the form $\forall x_1 \forall x_2 ... \forall x_n \varphi$ (abbreviated as $\forall \vec{x} \varphi$), where φ is quantifier-free. A *sentence* of L is a formula of L without free (i.e. non-quantified) variables. A *theory* in L is a set of sentences of L. We denote formulae by lower case Greek letters, and theories by upper case Greek letters. $\forall$, when used in appropriate context, denotes the set of all $\forall$-sentences of L. *Atom* denotes the set of all atomic sentences of L. *nAtom* denotes the set of all negated atomic sentences of L. *Pos* denotes the set of all positive sentences of L. *Neg* denotes the set of all negative sentences of L. We denote the positive and the negative parts of the basic diagram of $\mathcal{M}$ by $\mathcal{D}^+(\mathcal{M})$ and $\mathcal{D}^-(\mathcal{M})$, respectively. A canonic structure for L is a first-order structure $\mathcal{N}$ such that the interpretation of C in $\mathcal{N}$ is 1-1 and *on* $\mathcal{N}$. By Σ_Γ we denote the set $Cn(\Sigma) \cap \Gamma$. In particular, Σ_L denotes the set of all first-order consequences of Σ within the language L.

Let $nAtom^*$ denote the class of all finite sets of negated atomic sentences of L, let Σ be a **consistent** $\forall$-theory, and let $Cond(\Sigma)$ denote the class of all these elements of $nAtom^*$ which are consistent with Σ. The following relation of weak negative forcing constitutes a technical tool used throughout the paper. This is a special case of the generalized model-theoretic forcing introduced and investigated in [Suc88]. (Cf. [Kei78] § 8 and [Mac78] § 3, or [Kei73], for a definition and basic properties of model-theoretic forcing.)

Definition 2.1 The relation $\overset{\mathrm{w}}{\Vdash}$ of *weak negative forcing* is defined inductively for all increments $p \in Cond(\Sigma)$ and sentences $\varphi \in L$ (Σ is an implicit parameter of this relation).

(i) If $\varphi \in Atom$ then $p \overset{\mathrm{w}}{\Vdash} \varphi$ iff $\Sigma \cup p \vdash \varphi$.

(ii) $p \overset{\mathrm{w}}{\Vdash} \neg\varphi$ iff $(\forall q \in Cond(\Sigma) : p \subseteq q) \neg (q \overset{\mathrm{w}}{\Vdash} \varphi)$.

(iii) $p \overset{\mathrm{w}}{\Vdash} \varphi \wedge \psi$ iff $p \overset{\mathrm{w}}{\Vdash} \varphi$ and $p \overset{\mathrm{w}}{\Vdash} \psi$.

(iv) $p \overset{\mathrm{w}}{\Vdash} \forall x \varphi(x)$ iff $(\forall c \in C)(p \overset{\mathrm{w}}{\Vdash} \varphi(c))$.

Other connectives are treated as appropriate abbreviations. □

Weak forcing defines an interesting consequence operation, investigated in [Mac78], (and other papers referenced there), [Suc84], [Suc85], [Suc86a], [Suc87], [Suc88], and [Suc89].

Definition 2.2 Operation ^{S} is defined on class of consistent subsets of $\forall$ by:

$$\Sigma^S = \{\varphi \mid 0 \overset{\mathrm{w}}{\Vdash} \varphi\}.$$ □

Operation ^{S} has the following properties.

Lemma 2.3 $$\Sigma^S = (\Sigma_{Pos \cap \forall})^S.$$

Proof in [Suc88], lemma 4.13. □

Lemma 2.4 For every $\Sigma \subseteq \forall$:

(i) $\Sigma \subseteq \Sigma^S$,

(ii) $\varphi \in \Sigma^S \supset \neg\varphi \notin \Sigma^S$,

(iii) Σ^S is closed under $\vdash$,

(iv) $(\Sigma^S)_{Pos\cap\forall} = \Sigma_{Pos\cap\forall}$,

(v) ^{S} is a maximal operation which satisfies (i)...(iv).

Proof [Suc88], theorem 5.12. □

^{S} is a well behaved consequence operation, however, a non-monotonic one.

Example 2.5 Let L contain a unary predicate symbol P. If $\Sigma = 0$ then $0 \overset{w}{\Vdash} (\exists x)(P(x))$, but if $\Sigma = \{(\forall x)\neg(P(x))\}$ then $0 \overset{w}{\nVdash} (\exists x)(P(x))$.

Proof. We have $0 \overset{w}{\Vdash} (\exists x)(P(x))$ iff (since $\exists$ stands for $\neg\forall\neg$) $0 \overset{w}{\Vdash} \neg(\forall x)\neg(P(x))$ iff (applying (ii), (iv), (ii) and (i) of definition 2.1) $(\forall p \in Cond(\Sigma))(\exists q \in Cond(\Sigma) : p \subseteq q)(\exists c \in C)(\Sigma \cup q \vdash P(c))$. If $\Sigma = 0$ then $Cond(\Sigma) = nAtom^*$. It suffices to take any $c_\circ \in C$ not appearing in p, and $q = \{P(c_\circ)\}$, to see that $0 \overset{w}{\Vdash} (\exists x)(P(x))$. If $\Sigma = \{(\forall x)\neg(P(x))\}$ then $(\forall c \in C)(\Sigma \vdash \neg P(c))$. Because all elements q of $Cond(\Sigma)$ are consistent with Σ, we conclude $(\forall c \in C)(\Sigma \cup q \nvdash P(c))$. Therefore, $0 \overset{w}{\nVdash} (\exists x)(P(x))$ in this case. □

We will use the following classical concepts, related to weak forcing, and their basic properties.

Definition 2.6 *of generic set.*
Set G is called generic (relative to Σ) iff:

(i) each finite subset of G is in $Cond(\Sigma)$

(ii) for each sentence $\varphi \in L$, $G \overset{w}{\Vdash} \varphi$ or $G \overset{w}{\Vdash} \neg\varphi$

where $G \overset{w}{\Vdash} \vartheta$ means: $(\exists \text{ finite } p \subseteq G)(p \overset{w}{\Vdash} \vartheta)$. □

Generic Set Theorem 2.7 For each $p \in Cond(\Sigma)$ there is a generic set G, with $p \subseteq G$.

Proof in [Kei78], thm. 8.5. □

Definition 2.8 Let G be a generic set relative to Σ. A generic model $\mathcal{M}(G)$ corresponding to G is any canonic structure for L, such that for every sentence $\varphi \in L$:

$$\mathcal{M}(G) \models \varphi \text{ iff } G \overset{w}{\Vdash} \varphi.$$ □

Generic Model Theorem 2.9 For each generic set G there is a unique (up to isomorphism) generic model $\mathcal{M}(G)$.

Proof in [Kei78], §8, thm. 8.6. □

The Completeness Theorem 2.10 For every sentence $\varphi \in L$ and every $p \in Cond(\Sigma)$:

$$p \overset{w}{\Vdash} \varphi \text{ iff } (\forall \text{ generic set } G \supseteq p)(\mathcal{M}(G) \models \varphi).$$

Proof by straightforward application of theorems 2.7 and 2.9. □

Finally, we will need the following technical fact.

Lemma 2.11 If $\Sigma \subseteq \forall$ then $(\Sigma_{Pos\cap\forall})_L = (\Sigma_{Pos})_L$.

Proof in [Suc88], lem. 7.1. □

3 The closed world assumption

In this paper we identify a deductive data base with a consistent $\forall$-theory Σ in language L. The consistent version cwa_S (introduced in [Suc87]) of Reiter's closed world assumption is defined as follows.

Definition 3.1 $\varphi \in cwa_S(\Sigma)$ **iff** $\varphi \in \forall$ and $(\Sigma \cup \{\varphi\})_{Pos} = \Sigma_{Pos}$. $\square$

In Section 5 we see that cwa_S is stronger than cwa in all cases cwa is consistent, and strictly stronger then generalized closed world assumption $GCWA$ of [Min82]. The following result is used in Section 4.

Lemma 3.2 For every consistent $\Sigma \subseteq \forall$,

$$cwa_S(\Sigma) = \Sigma^S \cap \forall.$$

Proof in [Suc88], corollary 7.5. $\square$

As an easy consequence of lemma 3.2 we obtain:

Corollary 3.3 Let $\Sigma \subseteq \forall$ be a consistent theory of L.

(i) $(cwa_S(\Sigma))_{Pos\cap\forall} = \Sigma_{Pos\cap\forall}$.

(ii) For each quantifier-free sentence φ of L,

$$cwa_S(\Sigma) \vdash \neg\varphi \text{ iff } (\forall p \in Cond(\Sigma))\neg(cwa_S(\Sigma) \cup p \vdash \varphi).$$ $\square$

Therefore cwa_S is $Pos\cap\forall$-conservative, and monotone with respect to increments from $Cond(\Sigma)$.

4 Minimal models

In this section we apply weak negative forcing to provide the minimal entailment with the $\forall$-complete syntactic characterization. For other less successful trials in this aspect, readers may refer to [BS84], [EMR85], [BH86], [Hin88], and [She88].

Definition 4.1 The binary relation $\preceq$ between first-order structures for language L is defined by

$$\mathcal{M} \preceq \mathcal{N} \text{ iff } M = N \text{ and } \mathcal{D}^-(\mathcal{N}) \subseteq \mathcal{D}^-(\mathcal{M}),$$

where M and N denote the universes of $\mathcal{M}$ and $\mathcal{N}$, respectively. The relation $\prec$ is the intersection of $\preceq$ and $\neq$. $\square$

Structure $\mathcal{M}$ for language L is called a $\preceq$-*minimal* model of Σ iff it is $\preceq$-minimal in the class of models of Σ. We call such $\mathcal{M}$ *minimal* iff it is canonic.

In this section, the central point of interest is the following:

Problem 4.2 Given a $\forall$-theory Σ, find a complete syntactic characterization of the set $Cn_{min}(\Sigma)$ of all sentences of L true in each minimal model of Σ. $\square$

We provide such a characterization for $Cn_{min}(\Sigma) \cap \forall$. Prior to this, let us state a theorem of [BS84] which guarantees a regular behavior of $\preceq$-minimal semantics for $\forall$-theories. This result is a proper generalization of lemma 3 of [She88]. The proof we present here is quoted from [Suc86b].

Theorem 4.3 If $\Sigma \subseteq \forall$ then the class of $\preceq$-minimal models of Σ is $\preceq$-dense in the class of models of Σ.

Proof. Let $\mathbf{K}$ be a chain (relative to $\prec$) of models of an $\forall$-theory Σ. Let $\mathcal{M} = \cap\mathbf{K}$. It may be easily verified by induction that for every quantifier-free formula φ and assignment $\vec{s}$ in $\mathcal{M}$, if for all $\mathcal{N} \in \mathbf{K}$, $\mathcal{N} \models \varphi[\vec{s}]$, then $\mathcal{M} \models \varphi[\vec{s}]$. Thus$(\forall \mathcal{N} \in \mathbf{K})(\mathcal{N} \models \forall \vec{x}\varphi)$ implies $\mathcal{M} \models \forall \vec{x}\varphi$, and hence $\mathcal{M} \models \Sigma$. The thesis is obtained by the Kuratowski - Zorn Lemma. $\square$

Corollary 4.4 If $\Sigma \subseteq \forall$ then the class of minimal models of Σ is $\preceq$-dense in the class of canonic models of Σ. $\square$

The following lemmas are used in proof of the Main Theorem 4.11.

Lemma 4.5 Every generic model of Σ_{Pos} is a model of Σ, also a generic one.

Proof. By lemma 2.4 (i), $\Sigma \subseteq \Sigma^S =$ (by lemma 2.3) $(\Sigma_{Pos\cap\forall})^S =$ (by lemma 2.11) $(\Sigma_{Pos})^S$, so every generic model of Σ_{Pos}, which by the completeness theorem 2.10 is also a model of $(\Sigma_{Pos})^S$, is a model of Σ. On the other hand, every generic set for Σ_{Pos} (= $\Sigma_{Pos\cap\forall}$ by lemma 2.11) is also generic for Σ. $\square$

Lemma 4.6 $\mathcal{M}$ is a minimal model for Σ iff for every $\varphi \in \mathcal{D}^+(\mathcal{M})$,

$$\Sigma \cup \mathcal{D}^-(\mathcal{M}) \vdash \varphi.$$

Proof. Implication to the left is obvious. For implication to the right assume that $\mathcal{M}$ is a minimal model for Σ and for some $\varphi \in \mathcal{D}^+(\mathcal{M})$, $\Sigma \cup \mathcal{D}^-(\mathcal{M}) \nvdash \varphi$. Let $\mathcal{N}$ be a discriminant (i.e. one which interprets distinct elements of C as distinct elements of N) model for $\Sigma \cup \mathcal{D}^-(\mathcal{M}) \cup \{\neg\varphi\}$. Because $\mathcal{M}$ is canonic and therefore discriminant, assume without loss of generality that $M \subseteq N$. By Łoś-Tarski Theorem (see e.g. [Bar78], Chap.A3, thm. 3.11) $\mathcal{N} \restriction M \models \Sigma \cup \mathcal{D}^-(\mathcal{M}) \cup \{\neg\varphi\}$. Of course, $\mathcal{N} \restriction M \prec \mathcal{M}$ - a contradiction. $\square$

Lemma 4.7 Let $\mathcal{M}$ be a minimal model of Σ. For each quantifier-free sentence $\varphi \in L$, $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \varphi$ iff $\mathcal{M} \models \varphi$.

Proof. We show by simultaneous induction that for each such φ,

(i) $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \varphi \supset \mathcal{M} \models \varphi$, and

(ii) $\neg(\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \varphi) \supset \mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \neg\varphi$.

1. Case of atomic φ. (i) $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \varphi$ means that $\Sigma \cup \mathcal{D}^-(\mathcal{M}) \vdash \varphi$. Since $\mathcal{M} \models \Sigma \cup \mathcal{D}^-(\mathcal{M})$ then $\mathcal{M} \models \varphi$.

   (ii) If $\neg(\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \varphi)$ then $\Sigma \cup \mathcal{D}^-(\mathcal{M}) \nvdash \varphi$, hence by lemma 4.6 $\varphi \notin \mathcal{D}^+(\mathcal{M})$, i.e. $\neg\varphi \in \mathcal{D}^-(\mathcal{M})$. This of course gives $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \neg\varphi$.

2. Case of $\varphi = \neg\psi$. (i) is obvious.

   (ii) $\neg(\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \neg\psi)$ implies by induction hypothesis $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \psi$, i.e. by definition 2.1 (iv), $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \neg\varphi$.

3. Case of $\varphi = \psi \wedge \vartheta$. (i) is obvious.

   (ii) $\neg(\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \psi \wedge \vartheta)$ by (iv) of definition 2.1 means $\neg(\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \psi)$ or $\neg(\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \vartheta)$, so by induction hypothesis, $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \neg\psi$ or $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \neg\vartheta$, i.e. by (iv) $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \neg\psi \vee \neg\vartheta$, which by (iv) is the same as $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \neg\varphi$. $\square$

Lemma 4.8 Every generic model of Σ is minimal.

Proof. Let $\mathcal{M}$ be a generic model of Σ, that is to say (by lemma 4.5), a generic model for Σ_{Pos}. Let $\varphi \in \mathcal{D}^+(\mathcal{M})$. By the definition 2.8 of a generic model, it means that $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \varphi$, that is to say, for some finite $p \subseteq \mathcal{D}^-(\mathcal{M})$, $p \overset{w}{\Vdash} \varphi$. Since φ is atomic it belongs to $Pos \cap \forall$, and then $p \overset{w}{\Vdash} \varphi$ means, by lemma 2.4 (iv), $\Sigma \cup p \vdash \varphi$. Hence $\Sigma \cup \mathcal{D}^-(\mathcal{M}) \vdash \varphi$. Application of lemma 4.6 completes the proof. $\square$

Lemma 4.9 If sentence $\varphi \in \forall$ is true in every generic model of Σ then $\varphi \in Cn_{min}(\Sigma)$.

Proof. Let $\varphi = \forall \vec{x} \psi(\vec{x})$, where $\psi(\vec{x})$ is a quantifier-free formula of L, and let $\mathcal{M}$ be a canonic minimal model for Σ with $\mathcal{M} \not\models \varphi$, i.e. $\mathcal{M} \models \exists \vec{x} \neg \psi(\vec{x})$. Canonicity of $\mathcal{M}$ gives $\mathcal{M} \models \neg \psi(\vec{c})$ for some $\vec{c} \in C$. By lemma 4.7 we obtain $\mathcal{D}^-(\mathcal{M}) \overset{w}{\Vdash} \neg \psi(\vec{c})$. which implies $(\exists p \in Cond(\Sigma))(p \overset{w}{\Vdash} \neg \psi(\vec{c}))$. Applying negation clause (ii) of definition 2.1 we get $\neg(0 \overset{w}{\Vdash} \neg\neg\psi(\vec{c}))$ or, by (iv) of this definition, $\neg(0 \overset{w}{\Vdash} \psi(\vec{c}))$. $\forall$-clause (iii) gives $\neg(0 \overset{w}{\Vdash} \forall \vec{x} \varphi(\vec{x}))$, hence by the completeness theorem 2.10 there exists a canonic model $\mathcal{N}$ of Σ with $\mathcal{N} \not\models \varphi$ $\square$

Lemma 4.10 For every sentence $\varphi \in \forall$, $\varphi \in Cn_{min}(\Sigma)$ iff $0 \overset{w}{\Vdash} \varphi$.

Proof by immediate applications of lemmas 4.8, 4.9, and theorem 2.10. $\square$

This way we have related minimalism and closed world assumption to each other.

Main Theorem 4.11 For each $\Sigma \subseteq \forall$ and each sentence $\varphi \in \forall$, the following conditions are equivalent:
(i) $\varphi \in cwa_S(\Sigma)$,
(ii) φ is true in all minimal models for Σ,
(iii) φ is true in all generic models for Σ,
(iv) $0 \overset{w}{\Vdash} \varphi$.

Proof. The equivalency of (i) and (iv) follows from lemma 3.2 and definition 2.2. The equivalency of (ii) and (iv) is given by lemma 4.10. The equivalency of (iii) and (iv) follows from completeness theorem 2.10. $\square$

Theorem 4.11 constitutes essential progress in characterizing minimal model semantics. In particular, it is stronger then analogical characterization of [She88], theorem 32, using Minker's *GCWA*. It is known that neither Reiter's cwa_R nor $GCWA$ prove exactly those $\forall$-sentences which are true in all minimal models of $\forall$-theory Σ.

5 Other closed world assumptions

If in the right side of the definition 3.1 "Σ" is substituted by "Σ_{Atom}", and "$\varphi \in \forall$" by "$\varphi \in (Atom \cup nAtom)$", then cwa_S is transformed into an equivalent definition of Reiter's cwa:

$$\varphi \in cwa(\Sigma) \textbf{ iff } \varphi \in (Atom \cup nAtom) \text{ and } (\Sigma_{Atom} \cup \{\varphi\})_{Pos} = (\Sigma_{Atom})_{Pos}.$$

Similarly, *GCWA* may be equivalently reformulated as:

$$\varphi \in GCWA(\Sigma) \textbf{ iff } \varphi \in (Atom \cup nAtom) \text{ and } (\Sigma \cup \{\varphi\})_{Pos} = (\Sigma)_{Pos}.$$

(*Proofs* are straightforward.)

The above observations make it visible that both *cwa* and *GCWA* restrict conclusions of cwa_S to atomic and negated atomic sentences. Moreover, *cwa* accepts only the atomic part of a data base as the set of premises. Knowing that cwa_S is $\forall$-complete with respect to minimal semantics of a data base, one can easily characterize the scope of such completeness for *cwa* and *GCWA*: *cwa* is $(Atom \cup nAtom)$-complete with respect to minimal semantics of atomic part of the data base, and *GCWA* is $(Atom \cup nAtom)$-complete with respect to entire data base.

6 Open problems

The syntactic characterization of the whole $Cn_{min}(\Sigma)$ remains one of the most intriguing open questions. Also the syntactic criterion of minimal modelability of Σ seems unknown. We believe that the following claim is true.

Conjecture 6.1 If the condition "$\varphi \in \forall$" was replaced by "$\varphi \in \forall \cup \exists$" in the definition 3.1 of cwa_S then the equivalence of (i) and (ii) in theorem 4.11 would hold for each minimally modelable Σ and $\varphi \in L$. $\square$

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