A Syntactic Characterization of Minimal Entailment[1]

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Abstract

This paper investigates a consistent versions cwa_S of sometimes inconsistent Reiter's closed world assumption cwa, proving (theorem 4.3) that for every $\forall$-sentence φ and for every $\forall$-theory Σ,

$$\varphi \in cwa_S(\Sigma) \textbf{ iff } \Sigma \vdash_{min} \varphi.$$

A relativized version of this characterization (theorem 7.3) remains valid if not all relations are subject to minimization. Moreover, the paper relates cwa_S to cwa and to Minker's generalized closed world assumption *GCWA*. Finally, a possibility of procedural semantics for cwa_S has been indicated.

1 Introduction

Reiter's closed world assumption *cwa* has been introduced in [Rei78]. It may be defined as follows.

> If an atomic sentence φ is not implied by the information contained in a data base then the negation of φ is asserted.

In some cases *cwa* has led to a contradiction when applied to indefinite data bases, where disjunctions of atomic sentences are allowed. E.g. $cwa(\varphi \vee \psi)$ entails both $\neg\varphi$ and $\neg\psi$, that is to say, $\neg(\varphi \vee \psi)$. Minker has proposed in [Min82] a weaker version *GCWA* of *cwa*. It asserts the negation of an atomic sentence φ only if for every positive (i.e. without appearance of negation) $\forall$-sentence ψ, which is non-derivable from a

[1] This paper generalizes results of [Suc89].

data base, $\varphi \vee \psi$ cannot be derived from that data base. It has been proved (cf. [She88], thm. 32.5) that $GCWA$ coincides with cwa in all data bases for which cwa is consistent.

Another version of closed world assumption, cwa_S, has been proposed in [Suc87]:

> A sentence φ is asserted if, and only if, it does not enlarge the set of positive consequences of the data base.

Minimal model semantics, based on Lyndon's relation of enlargement of [Lyn59], seems to reflect the intended meaning of closed world assumption. Therefore, a syntactic system completely characterizing this semantics may be recognized as a proper formulation of cwa. Finding such a system appears to be one of the main unsolved[2] problems in logical foundations of Artificial Intelligence. It turns out that cwa_S provides such characterization within the class of all first-order $\forall$-sentences (or, equivalently, within the class of all clauses). Other known solutions to the problem of characterization do not have this completeness property. In particular, $GCWA$ does not prove some composed quantifier-free sentences which are true in all minimal models for a data base.

2 Prerequisites from first-order logic

In the following sequel, we follow the standard terminology and notation of first-order model theory, which can be found in [Bar78], Chap. A2. We restrict ourselves to a first-order language L with logical connectives $\wedge, \vee, \neg, \forall$ and $\exists$ (all other connectives we treat as appropriate abbreviations). A formula φ is *atomic* iff no logical connective appears in φ. A formula φ of L is $\{=\}'$-*positive* ($\{=\}'$ denotes the set of all other than $=$ relation symbols of L) iff no relation symbol other than $=$ appears in φ within a scope of negation. An $\forall$-formula is a formula of the form $\forall x_1 \forall x_2 ... \forall x_n \varphi$ (abbreviated as $\forall \vec{x} \varphi$) where φ is quantifier-free. A *sentence* of L is a formula of L without appearances of free (i.e. non-quantified) variables. A *theory* in L is a set of sentences of L. We usually denote formulae by lower case Greek letters, and theories by upper case Greek letters. $\forall$, when used in appropriate context, denotes the set of all $\forall$-sentences of L, $Atom$ denotes the set of all atomic sentences of L, and $Pos_{\{=\}'}$ denotes the set of all $\{=\}'$-positive sentences of L.

First-order structures for L we usually denote by $\mathcal{A}$, $\mathcal{B}$, $\mathcal{M}$, $\mathcal{N}$, ..., and their domains by $A, B, M, N, ...$, respectively. The *satisfaction relation* is denoted by $\models$, i.e. $\mathcal{M} \models \varphi[s]$ means that φ is true in structure

[2] cf. [BS84], [EMR85], [BH86], [Hin88], and [She88].

$\mathcal{M}$ under assignment s of its free variables (formally, s is a function from variables of L into the domain M of $\mathcal{M}$). If φ is true in $\mathcal{M}$ under every assignment s then $\mathcal{M} \models \varphi$ is used instead. If F is a function symbol of L then $F^{\mathcal{M}}$ denotes the corresponding function in $\mathcal{M}$. Analogically, $R^{\mathcal{M}}$ denotes the relation in $\mathcal{M}$ corresponding to relation symbol R. We use $\mathcal{M} \subseteq \mathcal{N}$ iff $M \subseteq N$ and for every function symbol F of L, $F^{\mathcal{M}} = F^{\mathcal{N}} \upharpoonright M$, and for every relation symbol R of L, $R^{\mathcal{M}} = R^{\mathcal{N}} \upharpoonright M$, where "$\upharpoonright$" means "restricted to". $\mathcal{M} \models \Sigma$ is an abbreviation of: for all $\varphi \in \Sigma$, $\mathcal{M} \models \varphi$. It states that the structure $\mathcal{M}$ is a model of Σ. We denote the class of all models of Σ by $\mathbf{Mod}(\Sigma)$. We make an implicit use of the completeness theorem of first-order logic by applying $\Sigma \vdash \varphi$ in the sense of $\mathbf{Mod}(\Sigma) \subseteq \mathbf{Mod}(\varphi)$. We write $\Sigma \vdash_{\mathrm{K}} \varphi$ **iff** $\mathbf{K}$ is a class of first-order structures and $\mathbf{Mod}(\Sigma) \cap \mathbf{K} \subseteq \mathbf{Mod}(\varphi)$ (because of typographic problems we avoid using bold type style in subscripts). Moreover, by Σ_Γ we denote the set $Cn(\Sigma) \cap \Gamma$. In particular, Σ_L coincides with the set $Cn(\Sigma)$ of all first-order consequences of Σ within the language L.

Among others, we will need the following classic result from model theory of first-order logic.

Łoś - Tarski Theorem 2.1 Σ is equivalent to a $\forall$-theory **iff** for every $\mathcal{A}$ and $\mathcal{B}$, ($\mathcal{A} \models \Sigma$ and $\mathcal{B} \subseteq \mathcal{A}$) implies ($\mathcal{B} \models \Sigma$).

Proof e.g. in [Bar78], Chap A2, thm. 3.11. □

3 The closed world assumption

In this paper we identify a deductive data base with a consistent $\forall$-theory Σ in a first-order language L. The consistent version cwa_S (introduced in [Suc87] for purely relational languages) of Reiter's closed world assumption is defined as follows.

Definition 3.1 $\varphi \in cwa_S(\Sigma)$ **iff** $\varphi \in \forall$ and $(\Sigma \cup \{\varphi\})_{Pos_{\{=\}'}} = \Sigma_{Pos_{\{=\}'}}$. □

We will show in Section 5 that cwa_S is stronger than cwa in all cases cwa is consistent, and strictly stronger then generalized closed world assumption $GCWA$ of [Min82].

The following theorem shows that cwa_S forms a consistent consequence operation for conservative reasoning from $\{=\}'$-positive fragments of universally axiomatizable deductive data bases.

Theorem 3.2 *Let* $\Sigma \subseteq \forall$.

(i) $cwa_S(\Sigma)$ is consistent unless Σ is not.

(ii) If $\Pi \subseteq \forall$ and $\Pi_{Pos_{\{=\}'}} = \Sigma_{Pos_{\{=\}'}}$ then $\Pi \subseteq cwa_S(\Sigma)$.

Proof in [Suc87], theorem 4.1. $\square$

4 Minimal models

The need for some kind of minimal semantics does not seem new to the Philosophy of Science. One of well known articulations is the Ockham's Razor Principle: *Entia non sunt multiplicanda praeter necessitatem* ([Ock]). In Artificial Intelligence, the last decade abounded in struggles with syntactic media of minimization. However, the problem of syntactic characterization of minimal entailment, which seems to be one of the central issues in model theory of AI logic, was left without the solution[3]. In this section we hope to improve that situation, at least as far as $\forall$-fragment of L is concerned.

The relation $\preceq$ of partial ordering in the class of first-order structures for L is defined as follows:

> $\mathcal{M} \preceq \mathcal{N}$ **iff** $M = N$, and for every function symbol F of L, $F^{\mathcal{M}} = F^{\mathcal{N}}$, and for every atomic formula φ of L and every assignment s in $\mathcal{M}$, $\mathcal{M} \models \varphi[s]$ implies $\mathcal{N} \models \varphi[s]$.

(The relation $\preceq$ has been first introduced by Lyndon [Lyn59] under the name of *enlargement.*)

Structure $\mathcal{M}$ is a *minimal model* of Σ **iff** $\mathcal{M} \models \Sigma$, and for every $\mathcal{N} \models \Sigma$, if $\mathcal{N} \preceq \mathcal{M}$ then $\mathcal{N} = \mathcal{M}$. We denote the class of all minimal models of Σ by $\mathbf{Min}(\Sigma)$. Class $\mathbf{J}$ is called *dense* in class $\mathbf{K}$ iff for every $\mathcal{N} \in \mathbf{K}$ there is $\mathcal{M} \in \mathbf{J}$ with $\mathcal{M} \preceq \mathcal{N}$. Theory Σ is called *minimally modelable* iff $\mathbf{Min}(\Sigma)$ is dense in $\mathbf{Mod}(\Sigma)$, i.e. for every $\mathcal{M} \models \Sigma$ there exists a minimal $\mathcal{N} \models \Sigma$ with $\mathcal{N} \preceq \mathcal{M}$. We use $\Sigma \vdash_{min} \varphi$ as an abbreviation for $\Sigma \vdash_{Min(\Sigma)} \varphi$ and $Cn_{min}(\Sigma)$ as an abbreviation for $\{\varphi \in L \mid \Sigma \vdash_{min} \varphi\}$.

The following theorem will be used in the proof of our main result.

Lyndon Theorem 4.1 For every $\Sigma \subseteq L$, and every first-order structure $\mathcal{A}$ for L,

$$\mathcal{A} \models \Sigma_{Pos_{\{=\}'}} \textbf{ iff } (\exists \mathcal{B} \equiv \mathcal{A})(\exists \mathcal{M} \models \Sigma)(\mathcal{A} \subseteq \mathcal{B} \text{ and } \mathcal{M} \preceq \mathcal{B}).$$

Proof in [Lyn59], theorem 5. $\square$

A lemma from [BS84] guarantees a regular behavior of minimal model semantics in case of $\Sigma \subseteq \forall$ (the proof is quoted from [Suc86]).

[3] A special case of this problem for purely relational languages has been successfully attacked in [Suc89] with model-theoretic forcing.

Lemma 4.2 If $\Sigma \subseteq \forall$ then Σ is minimally modelable.

Proof. Let $\mathbf{K}$ be a chain (relative to $\preceq$) of models of an $\forall$-theory Σ. Let $\mathcal{M} = \cap\, \mathbf{K}$. It may be easily verified by induction that for every quantifier-free formula φ and assignment s in $\mathcal{M}$, if for all $\mathcal{N} \in \mathbf{K}$, $\mathcal{N} \models \varphi[s]$ then $\mathcal{M} \models \varphi[s]$. Thus $(\forall \mathcal{N} \in \mathbf{K})(\mathcal{N} \models \forall \vec{x}\varphi)$ implies $\mathcal{M} \models \forall \vec{x}\varphi$, and hence $\mathcal{M} \models \Sigma$. The Kuratowski - Zorn Lemma completes the proof. $\square$

Now, we are ready to prove our first main result.

Main Theorem 4.3 For every $\Sigma \subseteq \forall$ and $\varphi \in \forall$,

$$\varphi \in cwa_S(\Sigma) \textbf{ iff } \Sigma \vdash_{min} \varphi.$$

Proof. Implication to the right.
Assume $\varphi \in cwa_S(\Sigma)$. Let $\mathcal{A}$ be a minimal model for Σ. In particular, $\mathcal{A} \models \Sigma_{Pos_{\{=\}'}}$. Using $\varphi \in cwa_S(\Sigma)$ we obtain, by definition 3.1, $\mathcal{A} \models (\Sigma \cup \{\varphi\})_{Pos_{\{=\}'}}$. By Lyndon Theorem 4.1 it means that there exist models $\mathcal{M}$ and $\mathcal{N}$ satisfying $\mathcal{A} \subseteq \mathcal{M}$, $\mathcal{A} \equiv \mathcal{M}$, $\mathcal{N} \preceq \mathcal{M}$, and $\mathcal{N} \models \Sigma \cup \{\varphi\}$. Let us note that for every sequence $\vec{a}$ of elements of A, the value of $F^{\mathcal{N}}(\vec{a})(= F^{\mathcal{M}}(\vec{a}) = F^{\mathcal{A}}(\vec{a}))$ is in A. Hence $\mathcal{N} \upharpoonright A$ is a valid first-order structure for L. So, let $\mathcal{B} = \mathcal{N} \upharpoonright A$. We have $\mathcal{B} \subseteq \mathcal{N}$, and therefore, by $\Sigma \cup \{\varphi\} \subseteq \forall$ and Łoś-Tarski Theorem 2.1, $\mathcal{B} \models \Sigma \cup \{\varphi\}$. On the other hand, $\mathcal{B} \preceq \mathcal{A}$. Therefore by minimality of $\mathcal{A}$, $\mathcal{B} = \mathcal{A}$. Thus $\mathcal{A} \models \varphi$.

Implication to the left.
Assume $\Sigma \vdash_{\mathrm{Min}(\Sigma)} \varphi$. Let $\mathcal{A} \models \Sigma$. By lemma 4.2, Σ is minimally modelable. So, let $\mathcal{B} \preceq \mathcal{A}$ be a minimal model for Σ. We have $\mathcal{B} \models \varphi$, that is to say, $\mathcal{B} \models \Sigma \cup \{\varphi\}$. Using Lyndon Theorem 4.1 again, we obtain $\mathcal{A} \models (\Sigma \cup \{\varphi\})_{Pos_{\{=\}'}}$. We have shown that $(\Sigma \cup \{\varphi\})_{Pos_{\{=\}'}} \subseteq \Sigma_L$, which obviously implies $(\Sigma \cup \{\varphi\})_{Pos_{\{=\}'}} \subseteq \Sigma_{Pos_{\{=\}'}}$. Application of definition 3.1 completes the proof. $\square$

5 Other closed world assumptions

The relationships between cwa_S, cwa, and $GCWA$ will become clear after we reformulate their definitions. For that purpose we need the following fact.

Lemma 5.1 $\boxed{\forall}$ If $\Sigma \subseteq \forall \cap L$ then $(\Sigma_{\forall \cap Pos_{\{=\}'}})_L = (\Sigma_{Pos_{\{=\}'}})_L$.

Proof. Since $\Sigma_{\forall \cap Pos_{\{=\}'}} \subseteq \Sigma_{Pos_{\{=\}'}}$, it suffices to prove that $\Sigma_{\forall \cap Pos_{\{=\}'}} \models \Sigma_{Pos_{\{=\}'}}$. Let $\mathcal{A} \models \Sigma_{\forall \cap Pos_{\{=\}'}} (= \Sigma_{Pos_{\{=\}'}} \cap \forall)$. By Łoś - Tarski Theorem 2.1, there exists $\mathcal{B} \models \Sigma_{Pos_{\{=\}'}}$ with $\mathcal{A} \subseteq \mathcal{B}$. By Lyndon Theorem 4.1, there exist $\mathcal{M} \models \Sigma$, $\mathcal{B}' \models \Sigma_{Pos_{\{=\}'}}$, and $\mathcal{M} \models \Sigma$, such that $\mathcal{B} \subseteq \mathcal{B}'$, and $\mathcal{B}'$ is a homomorphic image of $\mathcal{M}$. Let A, B', and M be the domains of models $\mathcal{A}, \mathcal{B}'$, and $\mathcal{M}$, respectively. We have $A \subseteq B' = M$. Let $\mathcal{N} = \mathcal{M} \upharpoonright A$. Because $\Sigma \subseteq \forall$, by Łoś - Tarski Theorem, $\mathcal{N} \models \Sigma$. It is easily seen that $\mathcal{A}$ is a homomorphic image of $\mathcal{M}$. Using Lyndon Theorem again, we obtain $\mathcal{A} \models \Sigma_{Pos_{\{=\}'}}$. □

It allows us to put the definition of cwa_S in the following form.

$$\varphi \in cwa_S(\Sigma) \textbf{ iff } \varphi \in \forall \text{ and } (\Sigma \cup \{\varphi\})_{\forall \cap Pos_{\{=\}'}} = (\Sigma)_{\forall \cap Pos_{\{=\}'}}.$$

It turns out that if in the right side of the above statement "Σ" is substituted by "Σ_{Atom}", "$\varphi \in \forall$" by "$\varphi \in (Atom \cup nAtom)$", and "$Pos_{\{=\}'}$", by "$Pos$", then cwa_S is transformed onto an equivalent definition of Reiter's cwa, that is to say,

$$\varphi \in cwa(\Sigma) \textbf{ iff } \varphi \in (Atom \cup nAtom) \text{ and}$$
$$(\Sigma_{Atom} \cup \{\varphi\})_{\forall \cap Pos} = (\Sigma_{Atom})_{\forall \cap Pos}.$$

Similarly, $GCWA$ may be equivalently reformulated as

$$\varphi \in GCWA\ (\Sigma) \textbf{ iff } \varphi \in (Atom \cup nAtom) \text{ and}$$
$$(\Sigma \cup \{\varphi\})_{\forall \cap Pos} = (\Sigma)_{\forall \cap Pos}.$$

(*Proofs* are straightforward.)

The above observations make it clear that both cwa and $GCWA$ restrict conclusions of cwa_S to atomic and negated atomic sentences. Moreover, cwa accepts only the atomic part of a data base as the actual set of premises. Knowing that cwa_S is $\forall$-complete with respect of minimal model semantics of a data base, one can easily characterize the scope of such completeness for $GCWA$ and cwa: $GCWA$ is $(Atom \cup nAtom)$-complete with respect to minimal model semantics of the data base, and cwa is $(Atom \cup nAtom)$-complete with respect to minimal model semantics of its atomic part; both under necessary assumption that L does not contain the equality symbol $=$ (in such a case $Pos_{\{=\}'} = Pos$). Consequently, a modification $WGCWA$ of $GCWA$ (cf. [LMR88] for details) is not $(Atom \cup nAtom)$-complete with respect to minimal model semantics of the data base, simply because $WGCWA$ does not coincide with $GCWA$.

6 Example

Now, we provide an example which shows that cwa_S is closer to minimal model semantics than cwa_R and $GCWA$. We pick up a language L with two unary relation symbols S and T, and one constant symbol c. Moreover, we adopt $\Sigma = \{S(c) \vee T(c)\}$. Since the equality symbol

$=$ does not belong to L, without loss of generality we may restrict the semantics of L to finite models with four-element domains. In this case every model of Σ is isomorphic to triple $\langle C, C_S, C_T, a\rangle$, where C is the four-element domain of $\langle C, C_S, C_T, a\rangle$, C_S and C_T are the (unary) relations corresponding to symbols S and T, a is an element of C, and $a \in C_S \cup C_T$. For each $\varphi \in L$, $\Sigma \vdash \varphi$ iff φ is true in every four-element model of Σ (general methods of such reductions may be found in Ackermann's classic text [Ack54]). Every four-element *minimal* model of Σ is isomorphic to triple $\langle C, C_S, C_T, a\rangle$, where $C_S \cup C_T = \{a\}$, and $C_S \cap C_T = 0$. The class of all minimal models of Σ is dense in the class of all models of Σ.

Let $\varphi = \neg(S(c) \wedge T(c))$. We have $\varphi \in Cn_{min}(\Sigma)$. Theorem 4.3 yields $\varphi \in cwa_S(\Sigma)$. It follows from the definition of *GCWA* that *GCWA* may add to $Cn(\Sigma)$ only negations of atomic sentences. Hence $\varphi \notin GCWA(\Sigma)$. On the other hand, $\neg S(c) \wedge \neg T(c) \in cwa_R(\Sigma)$ because $\Sigma \not\vdash S(c)$ and $\Sigma \not\vdash T(c)$. Therefore cwa_R is inconsistent in this case.

7 Relativized closed world assumption

In this section we consider a more general case of minimal model semantics, relativizing it to a set Γ of relation symbols of L. We assume that Γ is a set of (not necessarily all) relation symbols of L, and that the equality symbol $=$ does not belong to Γ. We will use the following Γ-relativizations of the notions related to the positiveness and minimality.

A formula is Γ-*positive* iff it is equivalent to a formula which does not contain an appearance of a symbol of Γ in a scope of a negation. Pos_Γ denotes the set of all Γ-positive sentences of L.

The relation $\preceq_\Gamma$ of partial ordering in the class of first-order structures for L is defined as follows:

> $\mathcal{M} \preceq_\Gamma \mathcal{N}$ **iff** $M = N$, and for every function symbol F of L, $F^{\mathcal{M}} = F^{\mathcal{N}}$, and for every relation symbol R not in Γ, $R^{\mathcal{M}} = R^{\mathcal{N}}$, and for every atomic formula φ of L and every assignment s in $\mathcal{M}$, $\mathcal{M} \models \varphi[s]$ implies $\mathcal{N} \models \varphi[s]$.

It is easily seen that $\Gamma \subseteq \Gamma'$ implies $\preceq_\Gamma \subseteq \preceq_{\Gamma'}$.

The notions of Γ-*minimal model,* $\mathbf{Min}_\Gamma(\Sigma)$, Γ-*density*, and Γ-*minimal modelability* are defined appropriately by substituting $\preceq$ by $\preceq_\Gamma$ in the definitions of Section 4.

Our intention for introducing these Γ-relativizations is to exclude from the scope of minimization those relations which do not have their names in Γ. This gives us all the intended expressiveness of the predicate circumscription. Equivalently, extending L by distinct symbols ϑ for negations of relation symbols ϱ one does not want to minimize and adding axioms of the form $(\forall\vec{x})(\varrho(\vec{x}) \equiv \neg\vartheta(\vec{x}))$ gives us the same effect, however, for the price of unnecessary complications.

One can observe that if Γ contains all the relation symbols of L but the equality symbol $=$ then all Γ-relativized notions coincide with their corresponding Γ-free counterparts of Section 4. The definition of Γ-relativized cwa_S reads as follows.

Definition 7.1 $\varphi \in cwa_S^\Gamma(\Sigma)$ **iff** $\varphi \in \forall$ and $(\Sigma \cup \{\varphi\})_{Pos_\Gamma} = \Sigma_{Pos_\Gamma}$. $\square$

Let us note that the lemma 4.2 holds for Γ-relativized version of minimal modelability. However, to obtain the proof for Γ-relativized version of theorem 4.3 we need to strengthen Lyndon Theorem 4.1.

Theorem 7.2 For every $\Sigma \subseteq L$, and every first-order structure $\mathcal{A}$ for L,

$$\mathcal{A} \models \Sigma_{Pos_\Gamma} \textbf{ iff } (\exists\mathcal{B} \equiv \mathcal{A})(\exists\mathcal{M} \models \Sigma)(\mathcal{A} \subseteq \mathcal{B} \text{ and } \mathcal{M} \preceq_\Gamma \mathcal{B}).$$

Proof is easily obtained from the proof of theorem 5 in [Lyn59] by putting $Q_3 = \Gamma$ (it is essential that the equality symbol $=$ does not belong to Γ) instead of $Q_3 = \{=\}'$, where Q_3 is a parameter used in that proof. $\square$

It is a routine translation to transform the proof of theorem 4.3 onto the proof of its following generalization.

Main Theorem 7.3 For every $\Sigma \subseteq \forall$ and $\varphi \in \forall$,

$$\varphi \in cwa_S^\Gamma(\Sigma) \textbf{ iff } \Sigma \vdash_{\mathrm{Min}_\Gamma(\Sigma)} \varphi. \qquad \square$$

8 Open problems and conjectures

The syntactic characterization of the whole $Cn_{min}(\Sigma)$ remains one of the most intriguing open questions. Also the weakest syntactic criterion for minimal modelability of Σ seems unknown.

Since, in the first-order language of arithmetic with additional unary relation symbol S, *Peano's Axioms outside* S + *induction on* S + *minimality of* S prove all true arithmetic sentences relativized to S, our Π_2 construction based on non-increasing the set of Γ-consequences cannot be complete everywhere outside $\forall$. We believe, however, that the following claims are true.

Conjecture 8.1 If the condition "$\varphi \in \forall$" was replaced by "$\varphi \in \forall \cup \exists$" in the definition 3.1 of cwa_S then theorem 4.3 would hold for each minimally modelable Σ and $\varphi \in \forall \cup \exists$. $\square$

Conjecture 8.2 If the condition "$\varphi \in \forall$" was replaced by "$\varphi \in \forall \cup \exists$" in the definition 7.1 of cwa_S^Γ then theorem 7.3 would hold for each Γ-minimally modelable Σ and $\varphi \in \forall \cup \exists$. $\square$

An anonymous referee kindly pointed out the lack of procedural semantics for cwa_S. Certainly, it follows from the definition 3.1 that (the degree undecidability of) $cwa_S(\Sigma)$ is Π_1 relative to $Cn(\Sigma)$, or Π_2 relative to Σ, and seemingly there is no good reason why it should be less. On the other hand, all asymptotically decidable problems (in particular those ones asymptotically decidable by finite failure proof procedures) are Δ_2, which is a proper subclass of Π_2. Therefore the problem of asymptotic decidability of $cwa_S(\Sigma)$ relative to Σ appears both important and non-trivial.

Although, as it follows from analysis in Section 5, $GCWA(\Sigma)$ is a decidable in $cwa_S(\Sigma)$ subset of $cwa_S(\Sigma)$, which means that $GCWA$ is a simpler than cwa_S scheme of reasoning, the only immediate conclusion one can draw from the definition of $GCWA$ is that, similarly to $cwa_S(\Sigma)$, $GCWA(\Sigma)$ is Π_2 relative to Σ. This, probably, gave rise to introducing $WGCWA$, which has been demonstrated to be Δ_2 relative to Σ and complete with respect to a finite failure proof procedure $SLINF$ (see [LMR88] for details). As we have noted in Section 5, however, $WGCWA$ does not characterize minimal model semantics, even in class of atomic and negated atomic sentences in a purely relational language, and therefore it can not provide an asymptotic proof procedure for minimal entailment.

In our opinion this problem requires a different approach, which we briefly describe below. It has been demonstrated in [Suc88] that cwa_S is completely characterized by a generalized weak model-theoretic forcing. Moreover, an asymptotic "finite failure proof procedure" *forces* for $\forall\exists$-fragment of standard weak model-theoretic forcing has been presented in [Suc85], and proven to be recursive relative to set of atomic and negated atomic consequences of Σ. We believe that procedure *forces* may be extended over $\forall$-fragments of generalized weak model-theoretic forcing discussed in [Suc88] and its Γ-relativizations introduced in [ST]. If this is the case then such an extension constitutes a procedural semantics for cwa_S^Γ, and moreover the following supposition is true.

Conjecture 8.3 $cwa_S^\Gamma(\Sigma)$ is Π_1 relative to Γ and $Cn(\Sigma) \cap$ *Quantifier-free*. $\square$

The truthfulness of conjecture 8.3 in case of purely relational language L and $\Gamma = \{=\}'$ has been shown in [Suc87], theorem 6.1.

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