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Abstract—We focus on sensor networks that are deployed in challenging environments, wherein sensors do not always have connected paths to a base station, and propose a new data resilience problem. We refer to it as DRE$^2$: data resiliency in extreme environments. As there are no connected paths between sensors and the base station, the goal of DRE$^2$ is to maximize data resiliency by preserving the overflow data inside the network for maximum amount of time, considering that sensor nodes have limited storage capacity and unreplacable battery power. We propose a quadratic programming-based algorithm to solve DRE$^2$ optimally. As quadratic programming is NP-hard thus not scalable, we design two time efficient heuristics based on different network metrics. We show via extensive experiments that all algorithms can achieve high data resiliencies, while a minimum cost flow-based is most energy-efficient. Our algorithms tolerate node failures and network partitions caused by energy depletion of sensor nodes. Underlying our algorithms are flow networks that generalize the edge capacity constraint well-accepted in traditional network flow theory.

Keywords – Data resilience, integer quadratic and linear programming, network flows, wireless sensor networks.

I. INTRODUCTION

Background and Motivation. Data resilience refers to the ability of any network to recover quickly and to continue maintaining availability of data despite of disruptions such as equipment failure, power outage, or malicious attack. Due to resource constraint challenges of wireless sensor networks such as unreplacable battery power and limited storage capacity of sensor nodes [35], link unreliability and scarce bandwidth of wireless medium [41], and the inhospitable and harsh environments in which they are deployed [6, 7], sensor nodes are often prone to failure and vulnerable of data loss. Therefore, how to ensure that collected data reaches the base station reliably has been an active research since the inception of sensor network research. This research is usually named data resilience [20], reliable data transmission [27], or data persistence [23]. We use data resilience throughout the paper.

However, all the existing data resilience research in traditional sensor networks assumes that a base station is always available to collect data, and focuses on how to encode and transmit data to the base station reliably. In this paper, we instead study data resilience from a totally different angle – from emerging sensor network applications wherein a base station is not available to collect the data. Such applications include volcano and seismic sensor networks [26], underwater or ocean sensor networks [9, 28], and volcano eruption monitoring and glacial melting monitoring [11, 32].

There are two common features of these sensor networks. First, they are all deployed in inaccessible or inhospitable regions, or under extreme weather. Therefore it is not feasible to deploy data-collecting base stations with power outlets in or near the sensor field. Second, as they continuously collect large volumes of data for a long period of time without the base stations, sensory data generated thus has to be stored inside the network first and then being collected when data mules or mobile sinks visit the sensor field periodically [8, 30]. Lacking human intervention and maintenance due to extreme environments, these sensing applications must operate more resiliently and robustly than traditional sensor networks.

Data Resilience Against Sensor Storage Overflow. In this paper, we focus on data resilience against sensor storage overflow, wherein some sensor nodes are close to the events of interest and continuously generate large amounts of sensory data, thus depleting their storage spaces as data nodes; the part of data that cannot be stored locally is referred to as overflow data. Other sensor nodes that have available storage are storage nodes (sensor nodes that generate data but still have available storages are considered as storage nodes).

In order to prevent data loss, the overflow data at the data nodes must be offloaded to the storage nodes to be preserved, waiting to be collected by aforesaid uploading opportunities. Otherwise, data loss occurs and the data resiliency is not achieved. The storage nodes that finally store overflow data
Data resilience has been an active research since the inception of sensor network research. Ghose et al. [16] achieved resilience by replicating data at strategic locations in the sensor network. Ganesan et al. [14] constructed disjoint multipaths to enable energy efficient recovery from node failures. Recently, network coding techniques are used to recover data from failure-prone sensor networks. Albano et al. [5] proposed in-network erasure coding to improve data resilience to node failures. Kamra et al. [21] proposed to replicate data compactly at neighboring nodes using growth codes that increase in efficiency as data accumulates at the sink. As wireless sensor networks utilize sleeping mechanisms to conserve energy, which causes data availability problem, Xu et al. [38] proposed a dataset synchronization protocol in named data networking to achieve data resilience. However, all these research adopts the traditional sensor network model wherein base stations are always available near the networks, therefore do not target the data resilience problem studied in this paper.

In sensor network where the base stations are absent, data resilience is achieved by preserving data inside the network, with two lines of work. The first line is a sequence of system research [25, 37, 40] that designed cooperative sensor storage systems to improve the network storage utilization. The other line of work is our own research. We instead took an algorithmic approach and solving a variety of data offloading problems with different objectives in sensor networks from minimizing the total energy consumption [6, 35], maximizing the total priorities of preserved data [39] or the minimum remaining energy of destination nodes [19], replicating data packets in the events of node failures [6, 34], to overcoming the overall storage overflow [33]. However, none of them attempts to maximize the data resilience defined in this paper.

### III. Problem Formulation of DRE$^2$

#### Network Model
We model a sensor network as an undirected graph $G(V, E)$, where $V = \{1, 2, ..., n\}$ is the set of $n$ nodes, and $E$ is the set of $m$ edges. Two nodes are connected if their distance is within the sensor nodes' transmission range. There are $l$ data nodes denoted as $V_d = \{1, 2, ..., l\}$. Data node $i \in V_d$ has $d_i$ number of overflow data packets, each has size of $k$ bits. The rest $n - l$ nodes are storage nodes $V - V_d = \{l + 1, l + 2, ..., n\}$. We denote the total $a = \sum_{i=1}^{l} d_i$ overflow data packets as $D = \{D_1, D_2, ..., D_a\}$. Let the data node of $D_j$ be $d_n(j) \in V_d$. Let $m_j$ be the available free storage space at storage node $j \in V - V_d$, meaning that $j$ can further store $m_j$ data packets. We assume that $a \leq \sum_{j=l+1}^{n} m_j$; otherwise, data loss is inevitable and data resilience is not achieved. We leave the more general question of under which conditions that not all the data packets can be offloaded as a future work.

#### Augmented Energy Model
We augment the well-known first order radio model [18] for wireless energy consumption. When node $u$ sends a $k$-bit data packet to its one hop neighbor node $v$ over their distance $l_{u,v}$ meters, the transmission energy spent by $u$ is $E_u^T(v) = \epsilon_{elec} \times k + \epsilon_{amp} \times k \times l_{u,v}^2$, the receiving energy spent by $v$ is $E_v^R = \epsilon_{elec} \times k$. Here $\epsilon_{elec} = 100nJ/bit$ is the energy consumption per bit on the transmitter circuit.
Table I

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>The set of $n$ data nodes</td>
</tr>
<tr>
<td>$V_d$</td>
<td>$V_d = {1,...,l}$ is the set of $l$ data nodes, and $V' = V \setminus V_d = {l + 1,l + 2,...,n}$ is the set of storage nodes</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Number of overflow data packets from data node $i \in V_d$</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Storage capacity of storage node $j \in V - V_d$</td>
</tr>
<tr>
<td>$D$</td>
<td>$D = {D_1,D_2,...,D_a}$ is the set of all overflow data packets</td>
</tr>
<tr>
<td>$dn(j)$</td>
<td>The data node of $D_j$ in $D$</td>
</tr>
</tbody>
</table>

$E^i$ is the initial energy level of sensor node $i$.

$E^i_{\sigma}$ is the energy level of sensor node $i$ after offloading.

$E^i_{\sigma}(v)$ is the transmission energy spent by $u$ to transmit one packet to $v$.

$E^i_{\sigma}$ is the receiving power spent by, to receive one data packet.

$E^{\text{store}}$ is the energy consumed for storing a data packet.

$E^{\text{store}}$ is the energy consumed for storing a data packet.

$r$ is the data offloading function.

$P_j$ is the offloading path of data packet $D_j$ in $D$.

$\sigma_{i,j}$ is node $i$'s successor node in $P_j$.

$y_{i,j}$ is node $i$'s energy cost of offloading data packet $D_j$.

$\xi_i$ is the number of data packets stored at storage node $i$.

$e_{\text{amp}} = 100pJ/bit/m^2$ is the energy consumption per bit on the transmit amplifier.

This model does not take into account the energy consumption of storing (or writing) data packets into sensor nodes. As write operation costs 13.2μJ amount of energy in Toshiba 128MB flash [4] and we are dealing with large amounts of sensory data, we assume that energy consumption for storing data is non-negligible. We augment above first order radio model with storing energy cost as follows. When storing a data packet, a storage node $v \in V - V_d$ costs $E^{\text{store}} = e_{\text{store}} \times k$ amount of storing energy. Here we assume that $e_{\text{store}} = 100nJ/bit$ is the energy consumption of storing one bit onto the sensor storage. Let $E_{u,v} = E^u_{\text{store}}(v) + E^v_{\text{store}}$; we have $E_{v,v} = E_{u,v}$. Note a data node not only transmits all of its own data packets, but also can receive and transmit (i.e., relay) data packets for other data nodes. Meanwhile, a storage node can receive data packets from other nodes and then either transmits or stores them. Table I shows all the notations.

**Problem Formulation.** We define offloading function as $r: D \rightarrow V - V_d$, indicating that data packet $D_j \in D$ is distributed from its data node $dn(j) \in V_d$ to its destination node $r(j) \in V - V_d$. Let $P_j : dn(j),...,r(j)$, referred to as the offloading path of $D_j$, be the sequence of distinct sensor nodes along which $D_j$ is offloaded from $dn(j)$ to $r(j)$. Let $\sigma_{i,j}$ denote node $i$’s successor node in $P_j$. Let $y_{i,j}$ be node $i$’s energy cost of offloading data packet $D_j$, then

$$y_{i,j} = \begin{cases} E^i_{\sigma}(\sigma_{i,j}) & i = dn(j), \\ E^i_{\sigma} + E^i_{\sigma} & i = r(j), \\ E^i_{\sigma}(\sigma_{i,j}) & i \in P_j - \{dn(j),r(j)\}, \\ 0 & \text{otherwise}. \end{cases} \quad (1)$$

When $i$ is the data node of $D_j$, it costs transmission energy $E^i_{\sigma}(\sigma_{i,j})$; when it is the destination node of $D_j$, it costs both receiving energy $E^r_{\sigma}$ and storing energy $E^{\text{store}}_{\sigma}$; when it is a relaying node of $D_j$, it costs both receiving and transmission energy, the sum of which is $E^i_{\sigma}(\sigma_{i,j})$. Otherwise, node $i$ is not involved in $D_j$’s offloading thus costs zero amount of energy. Let $E_i$ denote sensor node $i$’s initial energy level and $E'_i$ its remaining energy level after all the $a$ data packets are offloaded, respectively. Then, $E'_i = E_i - \sum_{j=1}^a y_{i,j}, \forall i \in V$.

**Definition 1:** (Data Resilience Levels (DRLs)). Given a sensor network $G$ with $a$ data packets to be offloaded, its data resilience level (DRL), denoted as $D(G)$, is defined as the sum of remaining energy of the destination nodes of all the $a$ data packet; i.e., $D(G) = \sum_{j=1}^a E'_{r(j)}$. It is also the case that $D(G) = \sum_{i=1}^n \xi(i)\chi(i)$, where $\chi(i)$ is the number of data packets that are finally stored at storage node $i$. $D(G)$ indicates the network’s best achievable effort to preserve data packets, as the more energy of a storage node, the longer time its stored data can survive. The objective of DRE$^2$ is to find an offloading function $r$ and a set of offloading paths $P = \{P_1,P_2,...,P_a\}$, for each of the $a$ data packets, such that the DRL of the network is maximized, i.e., $\max_{r,P} D(G)$, under the energy constraint of sensor nodes: $E'_i \geq 0, \forall i \in V$ and the storage capacity constraint of storage nodes: $\{j \mid r(j) = i, 1 \leq j \leq a\} \subseteq m_i, \forall i \in V - V_d$. Note that in event-driven or real-time scenarios wherein new data nodes appear periodically (due to new events of interest), we can apply our algorithms in a time-slotted manner to maximize the data resilience of their newly generated data packets.

**Example 1:** Fig. 2(a) shows a linear sensor network with four nodes: 1 and 2 are data nodes, each has two overflow data packets; 3 and 4 are storage nodes, each has storage capacity of four and $E_4 \gg E_3$. To achieve maximum DRL, the optimal solution is to offload all the four data packets to node 4, even though it costs more energy than offloading to node 3. We use this example to illustrate our QP and ILP solutions next.

IV. **QUADRATIC PROGRAMMING (QP) SOLUTION**

To maximize the DRL of any given instance of DRE$^2$, the fundamental challenge is to find each data packet’s destination node as well as its offloading path from its data node to this destination node. Then we are able to compute the number of data packets each destination node stores as well as its remaining energy level, therefore calculating the DRL. In particular, we need to represent sensor energy levels, data packets offloaded, and storage capacities utilized such that they can be computed in a holistic manner – network flow modeling [29] is particularly suitable for such purpose.

**Graph Conversion.** To demonstrate the network flow technique, we first convert the sensor network graph $G(V,E)$ in Fig. 2(a) to a flow network $G'(V',E')$ in Fig. 2(b).

First, it replaces each undirected edge $(i,j) \in E$ with two directed edges $(i,j)$ and $(j,i)$ of capacities $+\infty$. Second, it splits node $i \in V$ into two nodes in-node $i'$ and out-node $i''$ and adds a directed edge $(i',i'')$ with capacity of $E_i$, the initial energy level of node $i$. Third, all incoming directed edges of node $i$ are incident on $i'$ and all outgoing directed edges of node $i$ emanate from $i''$. Therefore the two directed edges $(i,j)$ and $(j,i)$ are now changed to $(i'',j')$ and $(j'',i')$. Fourth, it connects a super source node $s$ to the in-node $i'$ of the data packets.
node $i \in V_d$ with an edge of capacity $d_i$, the number of data packets at data node $i$. Finally, it connects the out-node $i''$ of the storage node $i \in V - V_d$ to a super sink node $t$ with an edge of capacity $m_i$, the storage capacity of storage node $i$. We have $|V'| = 2n + 2$ and $|E'| = 2m + 2n$. This graph conversion technique is used in our previous work [19, 39] solving other variants of data offloading in sensor networks.

Rationale of above Conversion. The rationale of above conversion is fourfold. First, as the flows start from $s$ and end at $t$ in flow network $G'$, and $s$ connects to in-nodes of data nodes while $t$ connects to out-nodes of storage nodes, it “forces” the overflow data packets to offload from data nodes to storage nodes. Second, with the node-splitting and the initial energy levels now being capacities of newly created edges, it guarantees that each node cannot exceed its energy capacity. Third, with the $d_i$ and $m_i$ now being “encoded” as the capacities of edges connecting $s$ and $i'$ (for data nodes) and $i''$ to $t$ (for storage nodes), it makes sure that a data node cannot offload more overflow data packets than it has and a storage node cannot store more overflow data packets than its storage allows. Fourth, and most importantly, the energy consumption of each node and the DRL can now be accurately computed using the network flows in $G'$, as shown below.

Computing the DRL. Let $x_{i,j}$ be the amount of flows on directed edge $(i, j)$ in $G'$. Recall that although both data nodes and storage nodes can receive overflow data packets from other nodes, a data node must transmit all of them whereas a storage node can store some of them as long as its storage allows and transmit the rest. Besides, a data node must transmit all of its own overflow data packets to others. Fig. 3(a) and (b) visualize the flows in $G'$ that go through a data node and a storage node $i$ respectively. We refer to the edge $(i', i'')$ in $G'$ as sensor node $i$’s internal edge, as the amount of flows on $(i', i'')$ represents the amount of “traffic” that goes through $i$. For data node $i$, such traffic includes offloading its own packets, receiving packets from other nodes, and transmitting packets to others, as shown in Fig. 3(a). For storage node $i$, such traffic includes receiving packets from others, transmitting packets to others, and storing packets into its local storage, as shown in Fig. 3(b).

We have below observations about the flows, their incurred energy cost, and the flow conservations.

- **Observation 1 (for both data and storage nodes).** Given any node $i \in V'$ and any of its neighboring node $j$ (i.e., $(i, j) \in E'$), the number of data packets $i$ receives from $j$ is $x_{j'', i'}$, the number of data packets $i$ transmits to $j$ is $x_{i'', j'}$. Thus, node $i$'s total receiving energy cost is $E_{r}^t \times \sum_{j, (i, j) \in E'} x_{j'', i'}$, and its total transmission energy cost is $\sum_{j, (i, j) \in E'} E_r^t(j) \times x_{i'', j'}$.

- **Observation 2 (for data nodes).** For any data node $i \in V_d$, the number of its own data packets that it transmits is $x_{s,i'}$. As the data packets $i$ transmits are either its own or received from others, we have $x_{s,i'} + \sum_{j, (i, j) \in E'} x_{j'', i'} = \sum_{j, (i, j) \in E'} x_{i'', j'} = x_{i', i''}$, where $x_{i', i''}$ is the amount of flow on edge $(i', i'')$.

- **Observation 3 (for storage nodes).** For any storage node $i \in V - V_d$, the number of data packets it stores is $x_{i', i''}$ (recall $\xi(i)$ is the number of data packets that are finally stored at storage node $i$). As the data packets $i$ receives are either transmitted to other nodes or stored at $i$, we have $\sum_{j, (i, j) \in E'} x_{j'', i'} + x_{i', i''} = x_{i', i''}$. The total storing energy cost of $i$ is $E_{s}^t \times x_{i', i''}$.

Therefore the DRL can be represented as below:

1Note that it is not that $(i, j) \in E'$, as all the data nodes, storage nodes, and neighboring nodes are sensor nodes in $G$, not in $G'$.
\[
D(G) = \sum_{i=t+1}^{n} \left( \xi(i) \times E_i \right) \\
= \sum_{i \in V - V_d} \left( x_{v', t} \times \left( E_i - E_i^{r} \times \sum_{j: (i, j) \in E} x_{j', v'} - \sum_{j: (i, j) \in E} \left( E_{i}^{s}(j) \times x_{j', v'} - E_i^{s} \times x_{v', t} \right) \right) \right).
\]

**QP Formulation.** As \( D(G) \) is a concave quadratic expression, DRE\(^2 \) can be represented using below QP formulation (A):

\[
\begin{align*}
(A) \quad & \text{max } D(G) \\
\text{s.t.} \quad & x_{s, v'} = d_s, \quad \forall i \in V_d \\
& x_{v', t}^{i} \leq m_i, \quad \forall i \in V - V_d \\
& x_{v', t}^{i} + \sum_{j: (i, j) \in E} x_{j', v'}^{i} = x_{v', v''}^{i}, \quad \forall i \in V_d \\
& x_{v', v''}^{i} = \sum_{j: (i, j) \in E} x_{j', v''}^{i}, \quad \forall i \in V_d \\
& x_{v', v''}^{i} = \sum_{j: (i, j) \in E} x_{j', v''}^{i} + x_{v', t}^{i}, \quad \forall i \in V - V_d \\
& E_i^{r} \times \sum_{j: (i, j) \in E} x_{j', v'}^{i} + \sum_{j: (i, j) \in E} \left( E_{i}^{s}(j) \times x_{j', v'}^{i} \right) \\
& \leq E_i, \quad \forall i \in V_d \\
& E_i^{r} \times \sum_{j: (i, j) \in E} x_{j', v'}^{i} + \sum_{j: (i, j) \in E} \left( E_{i}^{s}(j) \times x_{j', v'}^{i} \right) + E_i^{s} \times x_{v', t}^{i} \leq E_i, \quad \forall i \in V - V_d
\end{align*}
\]

Eqn. 4 mandates that to achieve data resilience, data node \( i \) must be able to offload all its \( d_s \) number of data packets into the network. Inequality 5 shows the storage constraint of storage nodes. Eqns. 6 and 7 show the flow conservation for in-node and out-node of each data node, respectively. The combination of these two equations shows that the total number of packets transmitted by a data node equals the number of its own offloaded packets plus the number of its received packets (Observation 2). Eqns. 8 and 9 show the flow conservation for in-node and out-node of each storage node, respectively. The combination of them shows that the total number of packets received by a storage node equals the number of packets transmitted by it plus the number of packets stored locally by it (Observation 3). Inequalities 10 and 11 represent the energy constraint of data nodes and storage nodes respectively. Note that data nodes consume energy when receiving and transmitting data packets while storage nodes consume energy when receiving, transmitting, and saving data packets (Observations 1 and 3).

**Generalized Edge Capacity Constraint.** For either a data node or a storage node \( i \) and its corresponding internal edge \((i', i'')\), although Inequalities 10 and 11 enforce energy constraint on \( i \), they do not reveal the relationship between \( x_{v', v''}^{i} \), the amount of flows on edge \((i', i'')\), and \( E_i \), the capacity of this edge. Their relationship does not simply follow the traditional edge capacity constraint \( x_{v', v''}^{i} \leq E_i \), that the amount of flows on an edge is less than or equal to the capacity on that edge. Before exploring such relationship for data nodes and storage nodes, we first define generalized edge capacity constraint as below. We will then show another forms of Inequalities 10 and 11 indeed exhibit such generalized edge capacity constraint.

**Definition 2:** (Generalized Edge Capacity Constraint.)
In a flow network, given any edge \((u, v)\), let \( f(u, v) \) and \( cap(u, v) \) denote its amount of flows and capacity respectively. The generalized capacity constraint is defined as \( \sum_{k=1}^{f(u, v)} a_k \leq cap(u, v) \), where \( a_k \) is the weight for the \( k \)th flow on the edge. When \( a_k = 1 \) for all the flows, it becomes \( f(u, v) \leq cap(u, v) \), the traditional edge capacity constraint.

For a data node \( i \) with \( x_{v', v''}^{i} \) flows going through it, we need to figure out which node each flow goes to in order to compute the incurred energy of this flow (note that energy cost of receiving a packet does not depend on where the packet is from while energy cost of transmitting a packet depends on where the packet goes). As the number of \( i \)’s own offloaded data packet is \( x_{s, v'}^{i} \), the number of data packets it relays (i.e., receives and then transmits) is \( x_{v', v''}^{i} - x_{s, v'}^{i} \). Assume the QP (A) has computed that the \( k \)th flow, \( 1 \leq k \leq x_{v', v''}^{i} \), goes to node \( \gamma(k) \). Then the energy cost of \( i \) to offload its own \( x_{s, v'}^{i} \) data packets is \( \sum_{k=1}^{x_{s, v'}^{i}} E_{i}^{s}(\gamma(k)) \), and the energy cost of \( i \) to relay the rest \( x_{v', v''}^{i} - x_{s, v'}^{i} \) packets is \( \sum_{k=x_{s, v'}^{i}+1}^{x_{v', v''}^{i}} (E_{i}^{r} + E_{i}^{s}(\gamma(k))) \). Therefore Inequalities 10 is changed to:

\[
\sum_{k=1}^{x_{s, v'}^{i}} E_{i}^{s}(\gamma(k)) + \sum_{k=x_{s, v'}^{i}+1}^{x_{v', v''}^{i}} (E_{i}^{r} + E_{i}^{s}(\gamma(k))) \leq E_i.
\]

Similarly, Inequalities 11, which is for a storage node \( i \) with \( x_{v', v''}^{i} \) flows going through it and \( x_{v', t}^{i} \) of them going to \( t \) (i.e., \( i \) stores \( x_{v', t}^{i} \) data packets), is changed to

\[
\sum_{k=1}^{x_{v', t}^{i}} (E_{i}^{r} + E_{i}^{s}(\gamma(k))) + \sum_{k=x_{v', t}^{i}+1}^{x_{v', v''}^{i}} (E_{i}^{r} + E_{i}^{s}(\gamma(k))) \leq E_i.
\]

**Theorem 1:** Eqns. 12 and 13 show that edges \((i', i'')\) in \( G' \) follow generalized edge capacity constraint.

**Proof:** For Eqn. 12, let \( a_k = E_{i}^{s}(\gamma(k)) \) when \( 1 \leq k \leq x_{s, v'}^{i} \), and \( a_k = E_{i}^{r} + E_{i}^{s}(\gamma(k)) \) when \( x_{s, v'}^{i} + 1 \leq k \leq x_{v', v''}^{i} \), then Eqn. 12 becomes \( \sum_{k=1}^{x_{s, v'}^{i}} a_k \leq E_i \), which is the generalized edge capacity constraint following Definition 2. Similarly, for Eqn. 13, let \( a_k = E_{i}^{r} + E_{i}^{s}(\gamma(k)) \) when \( 1 \leq k \leq x_{v', t}^{i} \), and \( a_k = E_{i}^{r} + E_{i}^{s}(\gamma(k)) \) when \( x_{v', t}^{i} + 1 \leq k \leq x_{v', v''}^{i} \), then Eqn. 13 becomes \( \sum_{k=1}^{x_{v', v''}^{i}} a_k \leq E_i \).

**Corollary 1:** There exists a special energy model under which Eqns. 12 and 13 demonstrate traditional edge capacity constraint where the number of flows on an edge is less than or equal to the edge capacity.
Proof: Let \( a_k = 1, \forall k \) for Eqns. 12 and 13, we get
\[
E_i^1(\gamma(k)) = E_i^r + E_i^s(\gamma(k)) = E_i^r + E_i^s + E_i^f(\gamma(k)).
\]
Thus \( E_i^r = 0 \) and \( E_i^f(\gamma(k)) = E_i^s = 1 \); that is, receiving a packet costs zero energy while transmitting and saving a packet each costs one unit of energy.

Intuitively, Corollary 1 gives a special and simplified energy model where every node on a data packet’s offloading path (including the data node, any intermediate relaying node, and the destination node) costs one unit of energy. This corresponds to the traditional edge capacity constraint where one amount of flow consumes one unit of edge capacity. By generalizing this widely used constraint in flow network, our work augments the existing network flow model and could have a potential impact on its related theory.

Solving QP. QP can be solved by the classic Wolfe’s modified simplex method [13], which is based on solving a system of linear relations subject to complementarity conditions. There are many production QP solvers such as CGAL [1] and CPLEX [2]. We adopt CPLEX due to its performances. Besides, CPLEX can improve the efficiency of QP by allowing gap tolerance to find a feasible solution quickly (more in Section VI). As QP is NP-hard [15], we next design two time-efficient heuristic algorithms below and show via experiments that they perform close to the optimal QP solution.

V. HEURISTIC ALGORITHMS

To maximize DRLs, an intuitive solution is to offload data packets to nodes with initial high energy levels, defined below.

Definition 3: (High-Energy Storage Nodes.) High-energy storage nodes, denoted as \( V_h \), are the set of storages nodes with the highest initial energy levels that can store all the \( a \) data packets. More formally, we sort the \( n-\ell \) storage nodes \( V-V_d \) in non-ascending order of their initial energy level: \( E_{v_1} \geq E_{v_2} \geq \ldots \geq E_{v_{n-\ell}} \). Then the top \( k+1 \) nodes \( \{v_1, \ldots, v_k, v_{k+1}\} \) where \( \sum_{i=1}^{k+1} m_{v_i} < a \leq \sum_{i=1}^{k} m_{v_i} \) is \( V_h \).

We design two algorithms below that center around how to offload data packets to \( V_h \) in an energy-efficient manner.

Network-Based Algorithm. For each high-energy storage node \( i \in V_h \), Algo. 1 finds \( m_i \) data packets that are closest to \( i \) (in terms of energy consumption) and offloads them to \( i \) via the currently available shortest path. This takes place until all the \( a \) data packets are offloaded. It takes \( O(n^2) \).

Algorithm 1: Network-Based Algorithm.

Input: A sensor network \( G \) with \( m_i \), \( E_i \), data packets \( D \); Output: \( D(G) \);
1. Compute \( V_h = \{v_1, \ldots, v_k, v_{k+1}\} \);
2. for \((1 \leq i \leq k)\)
3. Find the \( m_i \) data packets that are closest to \( v_i \) and offload them to \( v_i \) via the current shortest path between each data packet and \( v_i \);
4. Update the energy levels of all the nodes on the path;
5. end for;
6. Offload each of the \( a - \sum_{i=1}^{k} m_{v_i} \) data packets to \( v_{k+1} \) via the current shortest path and update the energy levels;
7. Compute \( D(G) = \sum_{i=1}^{k+1} (E_i^r \times \xi(i)) \);
8. RETURN \( D(G) \).

Minimum-Cost-Flow (MCF)-Based Algorithm. Although Algo. 1 saves energy by offloading data packets to their closest nodes in \( V_h \), it does not consider global energy minimization in data offloading. Below we design Algo. 2 that minimizes total energy consumption in data offloading. It is a MCF-based algorithm [29] applied on another properly converted flow network \( G''(V'', E'') \) from the sensor network \( G(V, E) \). In MCF, each edge in the flow network has a capacity and a cost and the goal is to minimize the total cost of the flows.

Graph Conversion. The conversion from \( G \) to \( G'' \) is similar to the one of converting \( G \) to \( G' \) in Section IV including the node splitting, the edge connectivity, and the edge capacities, however with two differences. First, the super sink \( t \) only connects to the out-nodes \( i'' \) of the high-energy storage node \( i \in V_h \), as we focus on offloading data packets to \( V_h \). Second, we assign the costs of all directed edge \( i'' \) as \( E_{i,j} = E_i^f(j) + E_i^r \), the sum of node \( i \)'s transmitting energy to \( j \) and node \( j \)'s receiving energy, the costs of directed edges \( j'' \) as \( E_{j,i} = E_i^s(i) + E_j^r \), the sum of node \( j \)'s transmitting energy to \( i \) and node \( i \)'s receiving energy, and the costs of all other edges as zeros.

MCF ILP Formulation. With above transformation, the sensor network \( G(V, E) \) in Fig. 2(a) is now converted to a flow network \( G''(V'', E'') \) in Fig. 2(c). Let \( x_{i,j} \) and \( c_{i,j} \) be the amount of flows and cost on edge \((i,j) \in E'' \), respectively. Below shows the MCF ILP formulation (B) that minimizes the total cost in \( G''(V'', E'') \).

\[
\begin{align*}
\text{(B)} \quad & \min \sum_{(i,j) \in E''} x_{i,j} c_{i,j} \\
\text{s.t.} \quad & x_{s,i''} = d_i, \quad \forall i \in V_d \\
& x_{i'',t} \leq m_i, \quad \forall i \in V_h \\
& x_{s,i''} + \sum_{j:(i,j) \in E} x_{j'',i''} = \sum_{j:(j,i) \in E} x_{j'',j''}, \quad \forall i \in V_d \\
& \sum_{j:(i,j) \in E} x_{j'',j''} = \sum_{j:(j,i) \in E} x_{j'',i''} + x_{i'',t}, \quad \forall i \in V_h \\
& E_i^r \times \sum_{j:(i,j) \in E} x_{j'',i''} + \sum_{j:(j,i) \in E} (E_i^f(j) \times x_{i'',j''}) \leq E_i, \\
& \forall i \in V_d \\
& E_i^r \times \sum_{j:(i,j) \in E} x_{j'',i''} + \sum_{j:(j,i) \in E} E_i^f(j) \times x_{i'',j''} \leq E_i, \\
& \forall i \in V_h \\
& \sum_{j:(i,j) \in E} x_{j'',i''} \leq \sum_{j:(j,i) \in E} x_{j'',j''}, \quad \forall i \in V - V_d - V_h \\
& E_i^r \times \sum_{j:(i,j) \in E} x_{j'',i''} + \sum_{j:(j,i) \in E} E_i^f(j) \times x_{i'',j''} \leq E_i, \\
& \forall i \in V - V_d - V_h
\end{align*}
\]
The Constraints 15-20 are similar to those in QP (A), except that Constraints 16, 18, and 20 are now applied on $V_h$, as only high-energy storage nodes $V_h$ can store data packets. Finally, we add two more constraints viz. Equation 21 and Inequality 22 to respectively address the flow conservation and energy constraint of all the storage nodes that are not in $V_h$. Algo. 2 below calls ILP (B) as a subroutine:

**Algorithm 2: MCF-Based Algorithm.**

*Input:* A sensor network $G$ with $m_i, E_i,$ and $D$;
*Output:* $D(G)$;  
1. Compute $V_h = \{v_1, ..., v_k, v_{k+1}\}$;  
2. Convert $G(V, E)$ to flow network $G''(V'', E'')$;  
3. Compute ILP (B) on $G''$;  
4. Compute $D(G) = \sum_{i=1}^{n} (E_i \times \xi(i))$;  
5. RETURN $D(G)$.

**Theorem 2:** Algo. 2 achieves minimum energy consumption in offloading a data packets to nodes in $V_h$.

**Proof:** We give a proof sketch due to space constraint. By applying MCF algorithm on $G''$, it guarantees that all the $a$ overflow data packets are offloaded to $V_h$ with minimum total energy cost while respecting the storage and energy constraints of sensor nodes.

**Solving MCF.** We implement MCF ILP using CPLEX [2]. MCF can also be solved efficiently and optimally by combinatorial algorithms such as scaling push-relabel proposed by Goldberg [17]. Its time complexity is $O(a^2 \cdot b \cdot \log(a \cdot c))$, where $a$, $b$, and $c$ are number of nodes, number of edges, and maximum edge capacity in the flow network, respectively.

All QP, Network- and MCF-based algorithms are fault-tolerant as they find the offloading paths in spite of node failures and network partitions caused by energy depletion of sensor nodes. We show that our fault-tolerant algorithms still achieve high DRLs and energy-efficiency in Section VI.

### VI. Performance Evaluation

We compare the performance of different algorithms viz. QP-based (referred to as QP), Network-Based (referred to as Network), and MCF-Based (referred to as MCF). We write our own simulator in Java that takes as input the sensor network instance including the network topology, initial energy levels of each sensor node $E_i$, the data nodes and their overflow data packets $d_i$, the storage nodes and their storage capacities $m_j$, and generates various output files that conform to the format needed for CPLEX execution. We consider both small scale networks of 50 sensor nodes (10 of them are data nodes) and large scale of 100 nodes (20 of them are data nodes). The sensor nodes are uniformly distributed in a region of $1000\times 1000$ m each. Each data node generates some number of overflow data packets, each of $512B$, that to be offloaded into the network. For initial energy levels, we consider both varying model, where different sensors have different initial energy levels, and uniform model, where all sensor nodes have the same initial energy levels. Transmission range is $250m$. In all plots, each data point is an average over ten runs, in each of which a different sensor network instance is generated. For fair comparisons of different algorithms, however, we use the same sensor network instance in the same run. The error bars indicate $95\%$ confidence intervals.

**Small-scale Comparison.** As QP takes time to run, we compare them in small scale of 50 nodes with 10 data nodes. The initial energy levels are random numbers in $[2000\mu J, 4000\mu J]$. Fig. 4(a) varies $d_i$ from 5, 10, 15, to 20 while setting $m_j$ as 5. It shows that with the increase of $d_i$, the DRLs achieved by all algorithms increase, as more data packets are now stored in storage nodes. QP always achieves slightly higher DRLs than MCF, while MCF higher than Network most of the time. As QP is optimal and all three perform close, it demonstrates the efficacy of all three algorithms in achieving data resilience. Fig. 4(b) shows the total energy consumptions of three algorithms, among which MCF has the smallest. QP costs the most energy, though, as it sometimes detours (instead of following the shortest energy path) from sources to destinations in order to achieve high DRLs. Fig. 5 varies $m_j$ while fixing $d_i = 10$. It shows again that QP achieves highest DRLs while MCF consumes the least amount of energy.

<table>
<thead>
<tr>
<th>Gap Tolerance (%)</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>30</th>
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<tr>
<td>Execution Time (sec)</td>
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<td>11.98</td>
<td>13.44</td>
<td>15.92</td>
</tr>
<tr>
<td>Network</td>
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<td>182.53</td>
<td>8.27</td>
<td>7.89</td>
<td>7.98</td>
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<td>MCF</td>
<td>1034.27</td>
<td>160.98</td>
<td>22.64</td>
<td>22.8</td>
<td>28.32</td>
</tr>
</tbody>
</table>

**TABLE II**

**Investigating Tolerance Gap of the QP in CPLEX.**

#### Large-scale Comparison.** Next we compare the algorithms in larger scale of 100 sensors, 20 of them are data nodes. We find that CPLEX cannot finish computing an instance of QP after more than 13 hours, thus decide to resort to its gap tolerance technique to improve its execution time.

Gap Tolerance of the QP. CPLEX [2] can be parameterized using a percentage value called *gap tolerance*, wherein
CPLLEX stops once it finds a feasible solution within this percent of optimal. We first investigate the tradeoff between the execution time and solution quality of different gap tolerance values. Table II records the CPLEX execution time for different gap tolerances between 2% and 30%, for three randomly generated networks. As 2% can take more than half an hour, we choose the next value of 3% as the gap tolerance for the QP; i.e., for the rest comparisons the QP always achieves at least 97% of the optimal DRLs.

Fig. 6 compares the three algorithms by varying $d_i$ from 50, 75, 100, to 125 with $m_j = 50$ and $E_i = 2500\mu J$. We observe that even with gap tolerance, QP still achieves slightly larger DRLs with much higher energy cost than the Network and MCF do, as it focuses on high DRLs and not on the energy efficiency to achieve them. It also shows that the energy cost of QP decreases when $d_i$ increases from 100 to 125. This is rather counter-intuitive, as offloading more data packets should cost more energy. Our conjecture is that when there are multiple routes to offload a data packet without affecting the DRL achieved, the QP randomly chooses one to save execution time. When the network has more data packets to offload, however, choosing such a path randomly could negatively affect the DRL maximization. As such, the QP chooses more energy-efficient offloading paths thus decreasing the energy cost. Fig. 7 varies $m_j$ and shows that the difference of DRLs by different algorithms seem to increase when increasing the storage capacity. As high energy nodes have more spaces to store data packets with increasing of $m_j$, the QP, being optimal, does a better job of utilizing the available spaces in order to maximize the DRL.

Investigating Fault-Tolerance. Finally we investigate the fault-tolerance of the three algorithms by finding the resultant dead nodes (i.e., nodes with depleted energy). We randomly generate one sensor network of 100 nodes, 20 of them are data nodes with their energies more quickly. For QP, with the increase of $d_i$, the number of dead nodes along the way. It stops until at least one of the algorithms fails to offload all the data packets. Fig. 8(a) sets $m_j = 50$ and shows that the algorithms can tolerate up to 11 dead nodes. However, as QP focuses more on DRLs and less on energy costs, it has more dead nodes than the other two most of the time. There is an interesting observation that when decreasing $E_i$, number of dead nodes increases in Network and MCF while decreases in QP. This is understandable for Network and MCF, as less initial energy levels contribute to more dead nodes in offloading the same amount of data packets. For QP, however, when $E_i$ gets low, in order to maximize the DRL, it distributes the data more evenly into the network, thus reducing the number of dead nodes. Fig 8(b) decreases $m_j$ to 13, at which point the network is almost full after data offloading. In contrast to Fig 8(a), there is no dead nodes for Network when $E_i$ is between 1200$\mu J$ and 800$\mu J$. When decreasing $m_j$, more storage nodes participate in the data offloading process, thus energy consumptions per node gets smaller, resulting in smaller number of dead nodes. Note that when $E_i = 200\mu J$, Network cannot offload all the data packets thus does not appear in the plot.

Last, Fig. 9(a) and (b) study the fault-tolerance of algorithms at $E_i = 1200\mu J$ by varying $d_i$ and $m_j$ respectively. It is interesting to notice that with the increase of $m_j$, the number of dead nodes increases for both Network and MCF while decreasing for QP. For Network and MCF, as the number of destination nodes gets smaller with increase of $m_j$, less number of them participate in the data offloading process, depleting their energies more quickly. For QP, with the increase of $m_j$ it can distribute data packets to nodes more evenly (by taking longer time), thus reducing the number of dead nodes.

VII. CONCLUSION AND FUTURE WORK

We solved a new data resilience problem that uniquely arises from emerging sensor network applications deployed in the
extreme environment. We designed a QP-based optimal algorithm and two time- and energy-efficient heuristic algorithms. Although sensor network research has been around for more than two decades, data resilience in our network setup has not been studied in any existing literature. We uncovered a generalized edge capacity constraint, wherein the consumed capacity on an edge is the linear combination of its flows. This generalizes the well-accepted edge capacity constraint in traditional network flows. One limitation of our approach seems to be that the network scenario including the data nodes and their overflow data packets are known beforehand. We believe this can be overcome by periodically executing our algorithms whenever new data nodes arise, thus solving the real-time version of this data resilience problem. As a future work, we plan to focus on a few specific topologies such as stars and trees, and investigate if optimal or approximate time-efficient algorithms exist. Finally, under which conditions such that not all the data packets can be offloaded and then how to achieve data resilience for such a fraction of data packets (i.e., event goodput [3]) remain two relevant new problems.

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REFERENCES