Proof of $\lfloor \text{Log2}[n] \rfloor + 1 = \lceil \text{Log2}[n+1] \rceil$

n is a positive integer here

$\lfloor \text{Log2}[n] \rfloor \leq \text{Log2}[n] < \lfloor \text{Log2}[n] \rfloor + 1$

$2^{\lfloor \text{Log2}[n] \rfloor} \leq 2^{\text{Log2}[n]} < 2^{\lfloor \text{Log2}[n] \rfloor + 1}$

$2^{\lfloor \text{Log2}[n] \rfloor} \leq n < 2^{\lfloor \text{Log2}[n] \rfloor + 1}$ (1)

$2^{\lfloor \text{Log2}[n] \rfloor} < n + 1 \leq 2^{\lfloor \text{Log2}[n] \rfloor + 1}$ (2)

[the right - hand inequality $\leq$ in (2) follows from the right - hand inequality $<$ in (1) because n and $2^{\lfloor \text{Log2}[n] \rfloor + 1}$ are an integers]

$\text{Log2}[2^{\lfloor \text{Log2}[n] \rfloor}] < \text{Log2}[n+1] \leq \text{Log2}[2^{\lfloor \text{Log2}[n] \rfloor + 1}]$

$\lfloor \text{Log2}[n] \rfloor < \text{Log2}[n+1] \leq \lfloor \text{Log2}[n] \rfloor + 1$

$\lceil \text{Log2}[n+1] \rceil = \lfloor \text{Log2}[n] \rfloor + 1$

A variant with k

Let $k = \lfloor \text{Log2}[n] \rfloor$

$\lfloor \text{Log2}[n] \rfloor \leq \text{Log2}[n] < \lfloor \text{Log2}[n] \rfloor + 1$

$k \leq \text{Log2}[n] < k + 1$

$2^k \leq 2^{\text{Log2}[n]} < 2^{k+1}$

$2^k \leq n < 2^{k+1}$ (3)

$2^k < n + 1 \leq 2^{k+1}$ (4)

[the right - hand inequality $\leq$ in (4) follows from the right - hand inequality $<$ in (3) because n and 2^{k+1} are an integers]

$\text{Log2}[2^k] < \text{Log2}[n+1] \leq \text{Log2}[2^{k+1}]$

$k < \text{Log2}[n+1] \leq k + 1$

$\lceil \text{Log2}[n+1] \rceil = k + 1$

$\lceil \text{Log2}[n+1] \rceil = \lfloor \text{Log2}[n] \rfloor + 1$

Optional Note

For any positive real number x such that
$2^i - 1 < x < 2^i$ \.1e
for some integer i,
the equality

$\lceil \text{Log2}[x+1] \rceil = \lfloor \text{Log2}[x] \rfloor + 1$

is false.

For all other positive real numbers x, the equality

$\lceil \text{Log2}[x+1] \rceil = \lfloor \text{Log2}[x] \rfloor + 1$

is true.

In[10]:=
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Plot[Tooltip[{Floor[Log2[x]] + 1 + 0.04, Ceiling[Log2[x + 1]]}],
 {x, 0, 16}, PlotTheme → "Classic"]
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Out[10]=