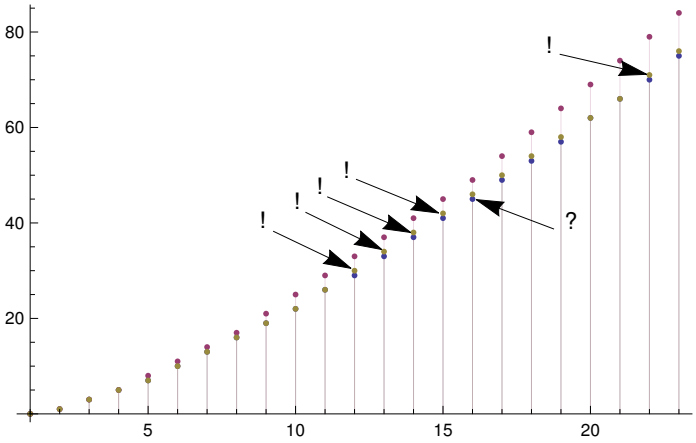
Graph of numbers of comps of sorting programs and the information - theoretic lower bound

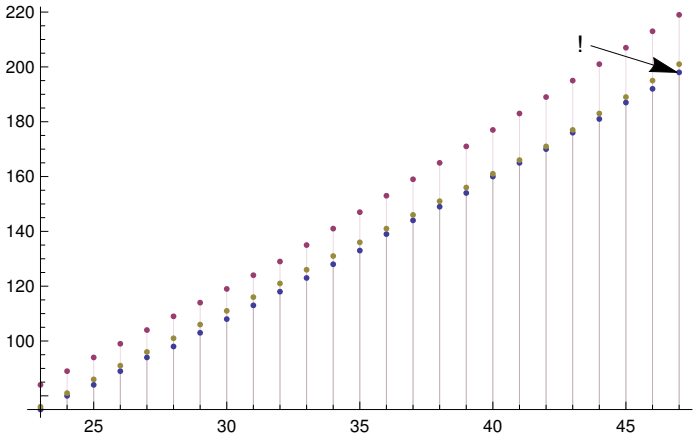
1. Worst case

Lower bound : $\lceil \lg n! \rceil$

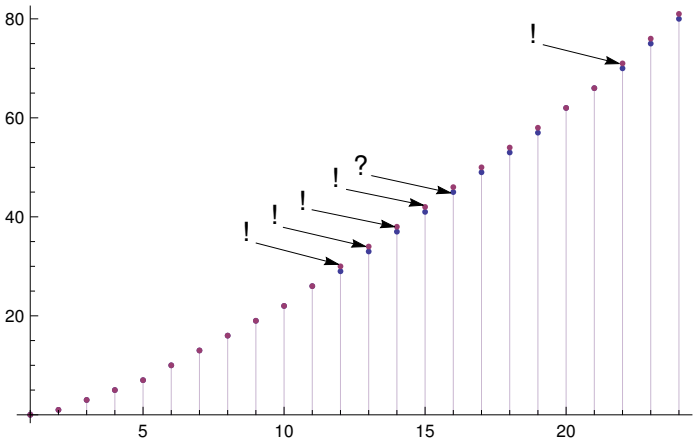
```
DiscretePlot[
 Tooltip[{⌈Log2[n!]⌉, n ⌈Log2[n]⌉ - 2^⌈Log2[n]⌉ + 1, Sum[⌈Log2[3/4 k]⌉, {k, 1, n}]}], {n, 1, 23}]
```



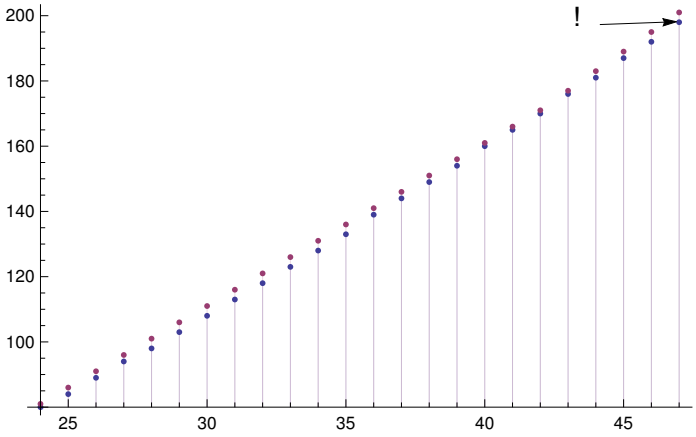

```
DiscretePlot[
 Tooltip[{⌈Log2[n!]⌉, n ⌈Log2[n]⌉ - 2^⌈Log2[n]⌉ + 1, Sum[⌈Log2[3/4 k]⌉, {k, 1, n}]}], {n, 23, 47}]
```



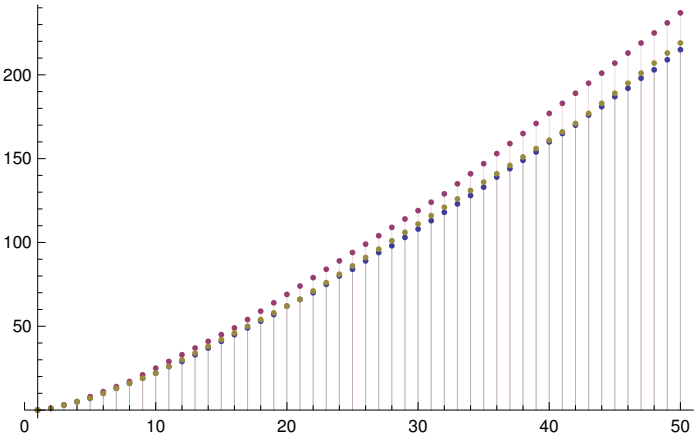

```
DiscretePlot[Tooltip[{⌈Log2[n!]⌉, Sum[⌈Log2[3/4 k]⌉, {k, 1, n}]}], {n, 1, 24}]
```



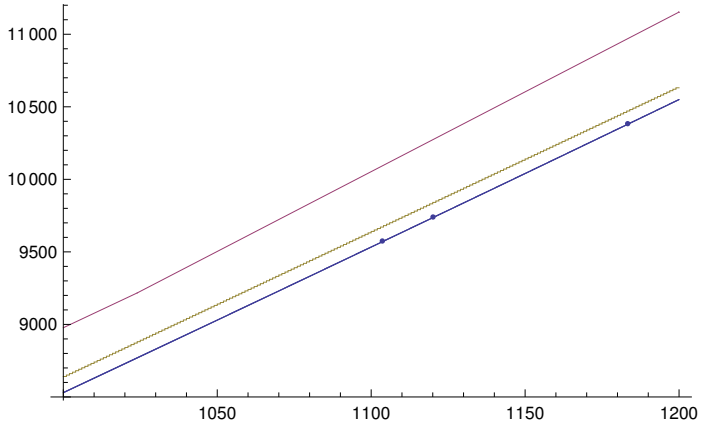

```
DiscretePlot[Tooltip[{⌈Log2[n!]⌉, ∑_{k=1}^{n} ⌈Log2[3/4 k]⌉}], {n, 24, 47}]
```




```
DiscretePlot[
 Tooltip[{⌈Log2[n!]⌉, n ⌈Log2[n]⌉ - 2^⌈Log2[n]⌉ + 1, ∑_{k=1}^{n} ⌈Log2[3/4 k]⌉}], {n, 1, 50}]
```



```
Plot[Tooltip[{⌈Log2[n!]⌉, n ⌈Log2[n]⌉ - 2^⌈Log2[n]⌉ + 1, ∑_{k=1}^{n} ⌈Log2[3/4 k]⌉}],
 {n, 1000, 1200}, PlotPoints → 2000]
```



Recall the approximate value of the lower bound on the worst - case :

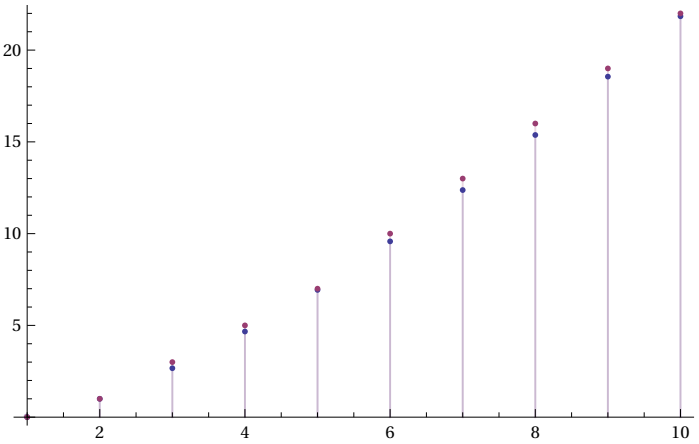
$$\left\lceil \left(n + \frac{1}{2}\right) \text{Log2}[n] - 1.44\, n + 1.33 \right\rceil$$

2. Average case

Lower bound : $\lg n! + \epsilon(n!)$

In[9]:=
```
DiscretePlot[Tooltip[{Log2[n!] + ϵ[n!], ⌈Log2[n!]⌉}],
 {n, 1, 10}, PlotTheme → "Classic"]
```

Out[9]=



where

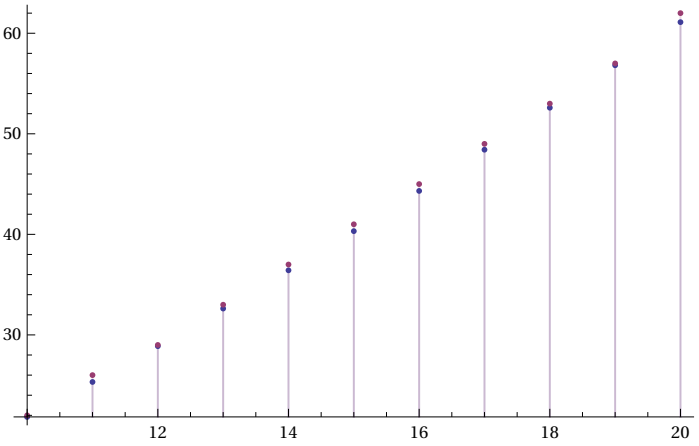
In[1]:=
```
β[x_] := 1 + x - 2^x

θ[x_] := ⌈x⌉ - x

ϵ[x_] := β[θ[Log2[x]]]
```

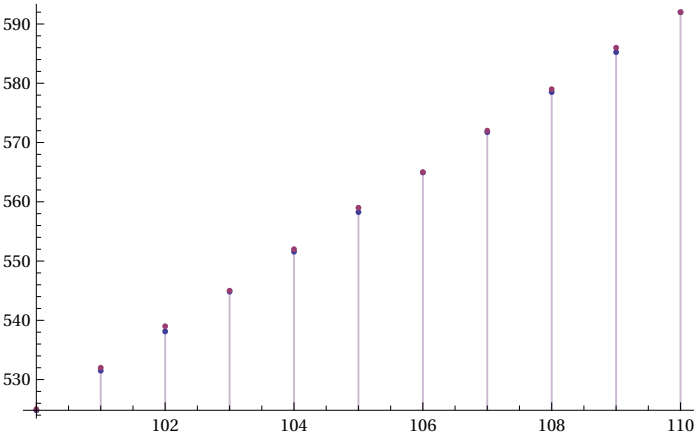
In[10]:=
```
DiscretePlot[Tooltip[{Log2[n!] + ϵ[n!], ⌈Log2[n!]⌉}],
 {n, 10, 20}, PlotTheme → "Classic"]
```

Out[10]=



In[11]:=
```
DiscretePlot[Tooltip[{Log2[n!] + ϵ[n!], ⌈Log2[n!]⌉}],
 {n, 100, 110}, PlotTheme → "Classic"]
```

Out[11]=



Recall approximate value of the lower bound on the average :

$$\left(n+\frac{1}{2}\right)\text{Log2}[n]-1.44\,n+1.33+\epsilon[n!]\approx$$

(the following is an underestimate)

$$\approx\left(n+\frac{1}{2}\right)\text{Log2}[n]-1.44\,n+1.33$$